

**Market adjustment and
investment determination:
A theoretical analysis of the
firm and the industry**

Ingemar Hansson

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MARKET ADJUSTMENT AND INVESTMENT DETERMINATION:
A THEORETICAL ANALYSIS OF THE FIRM AND
THE INDUSTRY

Ingemar Hansson

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PREFACE

This study emanates from research on the markets for houses and housing services under inflation, conducted with financial support from The Swedish Council For Building Research. This work evoked my interest in the behavior of a market with durable capital and its inter-relationship with the market providing investment goods. In combination with my interest in economic theory, this challenged me to conduct an analysis of market adjustment and investment determination on a more theoretical level. This research is presented in this book.

The study has benefitted from seminars on earlier drafts at The Econometric Society's Winter Symposium in Barcelona in January 1980 and at the Stockholm School of Economics in June 1980. I would like to acknowledge a debt of gratitude to the participants of these seminars notably Christopher Gilbert at Oxford Institute of Economics and Statistics, Karl Vind at University of Copenhagen, Peter Englund and Karl-Göran Mäler at the Stockholm School of Economics as well as Lars Svensson and Hans Tson Söderström at the Institute For International Economics Studies at the University of Stockholm.

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1 INTRODUCTION AND SUMMARY

1.1 Purpose of the Study

The purpose of this study is to examine a competitive market's adjustment to different types of shifts in factors exogenous to the market such as demand for output, supply of input, technology and interest rate. The study addresses a number of basic issues in economic analysis of market behavior. Since a market adjusts through capital changes, the theory of adjustment might equivalently be interpreted as a theory of investment. The study covers both interpretations.

The subject matter of this study is of interest for at least two reasons:

1. The theory developed here predicts and explains the effects of shifts in, for example, demand. The model might be applied to examine the effects of permanent or temporary shifts in demand caused by, for example, shifts in preferences, income or taxation. Similarly, the theory predicts and explains the effects of shifts in the supplies of inputs, in technology and in interest rates.

The model may be applied to analyze, for example, the effects of the sharp decline in demand for shipping services following the oil price increases in 1973-74. An analysis of the process of adjustment to such a major disturbance yields predictions on the development of prices and quantities in the market for shipping services and the market for ships. As another practical example, the theory predicts the effects of a change in taxation on the market for housing services and the market for houses. We might, for example, be particularly interested in the short- and long-run effects of a tax increase on housing construction. The adjustment path to these types of changes is analyzed in Chapters 3-5.

2. The results for the effects of basic shifts in exo-

genous factors also constitute building blocks of a theory explaining the determination of time paths for prices and quantities resulting for any given time paths for demand for output, supplies of inputs, technology and interest. Focusing on investment, the analyses of capital adjustment yield an investment theory, where the investment path is determined as a process of optimal capital adjustment for any expected paths for the exogenous factors. This is analyzed in Chapter 6.

The study starts with a comparative dynamics analysis of the effects of basic shifts in demand. Analogous results arise for other exogenous factors and for more complex cases.

We begin with the basic case of a market in an original stationary state with constant exogenous factors over time. We wish to address the following questions which arise if information emerges which leads to expectations of a permanent shift in demand at a future point of time:

1. How does a single firm in a competitive market respond to this shift in demand?
2. How does the industry, as an aggregate of the individual firms, respond to the shift?

These questions are answered for different types of basic permanent demand shifts. An unexpected shift is the special case when information on the shift emerges at the same time as the shift, i.e. information lead is zero. An expected shift corresponds to the case with a strictly positive information lead. Perfect foresight is the limiting case where the information lead approaches infinity.

1.2 Comparative Statics and Its Limitations

Textbooks in basic economics typically analyze a demand shift by means of comparative statics. The following diagram, Figure 1:1, illustrates the typical case where

the market's original equilibrium, $(X_0, P_{X,0})$ is determined by the original demand curve, $P_{X,0}(X)$, and the supply curve or the marginal cost curve, $MC(X)$. The marginal cost curve for the industry may be increasing as a result of decreasing returns to scale or increasing costs for inputs. A shift in the demand function from $P_{X,0}(X)$ to $P_{X,1}(X)$ then shifts the equilibrium from $(X_0, P_{X,0})$ to $(X_1, P_{X,1})$.

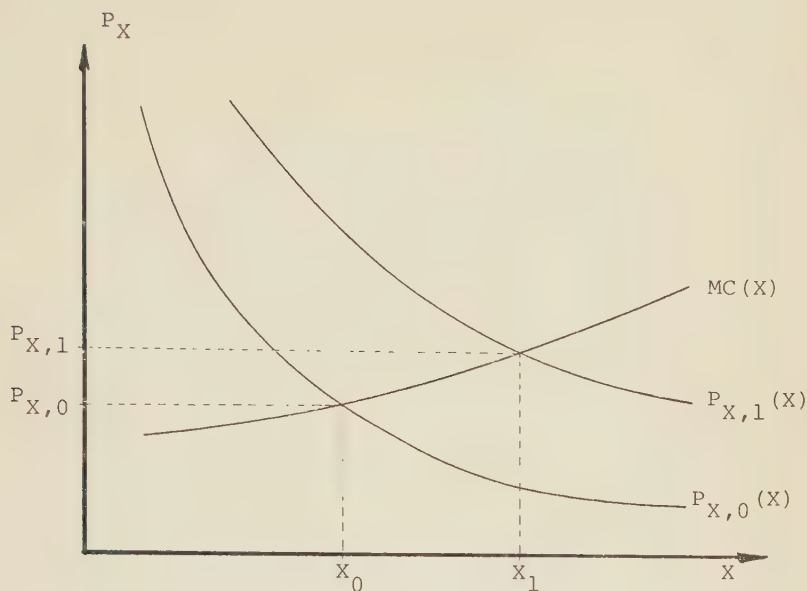


Figure 1:1 Comparative statics for a shift in demand.

A production process typically produces output using the services of a stock of durable inputs, i.e. using capital services. Therefore, the shift to the new long-run equilibrium in Figure 1:1 typically requires a capital adjustment. The stock of capital may, however, be changed only through a flow of net investment. If the supply conditions for investment goods preclude infinite investment for the industry there can be no instantaneous capital adjustment. The adjustment may instead be spread over a considerable period of time. As an example, an increase in demand for housing services requires an adjustment in the stock of housing through housing construction in excess of depreciation. This adjustment process might extend over several years.

The simple static model in Figure 1:1 determines the long-run effects of a permanent shift in demand. The comparative static analysis, however, provides no answer to the question of how prices and quantities are determined during the adjustment process. Moreover, the static analysis yields no information on the length of the adjustment period. The current study addresses both these questions.

1.3 The Importance of the Adjustment Process

1.3.1 Relevance for Short- and Medium-Run Effects

In order to predict and explain the short- and medium-run effects of expected and unexpected shifts in exogenous factors, we need a dynamic theory of the adjustment process between hypothetical long-run equilibrium positions. This is especially important for markets with slow adjustment. As an example, predictions of the comparative static analysis that a decrease in demand for shipping services leads in the long-run to lower output, lower price, and a smaller stock of ships with zero net investment, yield no information on the short- and medium-run effects of this demand shift. To obtain a fuller picture the theory should predict and explain price and quantity paths during the adjustment process.

This study aims to analyze adjustment in a competitive market to expected and unexpected shifts in exogenous factors. The study extends the comparative static results by including an analysis of the determination of price and quantity paths for output and inputs during the adjustment process between two long-run equilibrium positions.

A dynamic theory of adjustment is also required to examine the effects of temporary shifts in exogenous factors. The static theory for long-run equilibrium positions suggests that a temporary shift has no effect on the long-run position, without describing the short- and medium-run effects. In contrast, our dynamic

theory also provides answers for the effects of temporary shifts.

1.3.2 The Determinants of the Adjustment Speed

As a part of the analysis of the adjustment process we examine the determinants of adjustment speed between two long-run equilibrium positions for permanent shifts in exogenous factors. Analogous results arise for the degree of capital adjustment to temporary shifts in exogenous factors. If adjustment is very rapid, comparative static analysis yields reasonable approximations to the effects of permanent and temporary demand shifts. Slower adjustment increases the importance of an explicit analysis of the adjustment process. Such an explicit analysis is especially important for durable capital such as houses, roads, human capital, ships and durable machinery and equipment. This is analyzed further below.

The speed of adjustment is also important for specification of econometric models. Such models typically assume that the data employed exhibit some type of equilibrium positions. If most of the adjustment to the considered type of equilibrium are made within, say, 6 months, yearly data might be interpreted as showing approximate equilibrium positions. A slower adjustment process suggests the need for a longer, perhaps a considerably longer, minimum time period for each observation in time series as well as in cross section analysis. The frequent use of permanent instead of current income in econometric estimations illustrates the use of a longer time period in order to take account of slow adjustment. An analysis of the determinants of adjustment speed is, therefore, important in the specification of econometric models, especially in the determination of the proper degree of aggregation over time.

1.3.3 Theory of Adjustment as an Investment Theory

Since capital is adjusted through a flow of net invest-

ment, our study of capital adjustment might equivalently be interpreted as a theory of investment. Given such an interpretation the need for an analysis of the adjustment process is even more obvious. Classical capital theory as developed by Böhm-Bawerk (1891) and Wicksell (1901, 1934) is confined to long-run equilibrium positions with zero net investment. These models, therefore, provide no answer to the question of how investment might be affected by adjustment between long-run equilibrium positions.

If, in contrast to Wicksell (1934),¹⁾ this static model is interpreted as a model for short-run equilibrium too, net investment is determined by current changes in exogenous factors only. The model then yields instantaneous capital adjustment through infinite investment for the industry. Econometric estimations and casual empiricism suggest instead that capital adjusts successively over a period of time, implying that net investment takes place in response to past and expected changes in exogenous factors. The static model is, therefore, inappropriate for investment theory. Such a theory should, instead, be based upon an explicit analysis of the process of capital adjustment.

1.4. Interpretation of Capital and Investment

The term capital should be given quite a broad interpretation in this study. The term includes not only different types of physical capital but also, for example, different types of human capital and good-will

1) Wicksell stressed that his capital theory deals with long-run equilibriums and comparative statics for these solutions only:
 "As has already been pointed out, we shall assume stationary conditions as the foundation of our observations. This will not prevent us from considering changes in the quantities concerned, provided that we do not take into account the actual transition stage, which is a much more complicated problem, but assume that these changes have already become final, so that "static equilibrium" (a stationary state) is again restored." Wicksell (1934, p. 152).

for a firm or a brand name. As a result of the increasing degree of labor specialization and the decreasing degree of labor mobility, labor might be considered as a durable input called capital in the current model. The stock of labor is then changed through a flow of net hiring of new labor, corresponding to net investment in our model.¹⁾ The nondurable input in the current study is nevertheless denoted L and called labor in accordance with traditional terminology. The stock of durable input, K , might, of course, be interpreted as a stock of labor, in which case the nondurable input, L , may be inputs such as, raw material and energy.

1.5 Earlier Theories of Adjustment

1.5.1 Introduction

This section reviews the predictions of earlier theories of adjustment at the firm and the industry levels.

1.5.2 Short- and Long-Run Supply Curves

Most analyses of adjustment in basic economic theory as well as in applied economics are based on the Marshall (1890) - Viner (1931) concepts of short- and long-run supply curves.²⁾ In the short-run some, but not all, inputs are variable. The short-run marginal cost $SRMC(X)$, or the short-run supply curve is illustrated in Figure 1:2. An unexpected shift in demand causes a shift from the original short- and long-run equilibrium, $(X_0, P_{X,0})$, to the new short-run equilibrium position, $(X_s, P_{X,s})$, in Figure 1:2.

The higher output price, $P_{X,s}$, is assumed to induce increases in the capital stocks of existing firms and

1) Nadiri and Rosen (1969), Coen and Hickman (1970), Helliwell and Glorieux (1970), Schramm (1970), Nadiri (1972) and Kanis (1979) all estimate econometric models where labor as well as other inputs are treated as durable inputs with successive instead of instantaneous adjustments to shifts in exogenous factors.

2) See also Frisch (1950).

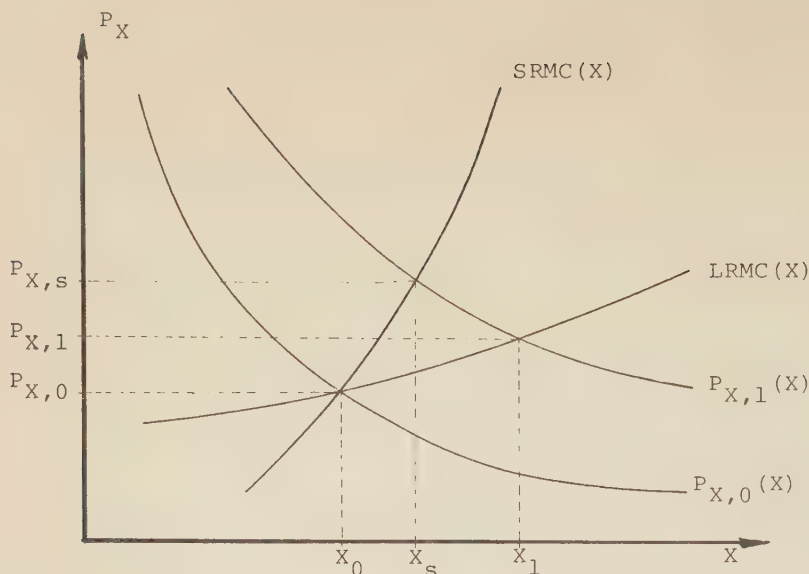


Figure 1:2 Adjustment analyzed by short- and long-run supply curves.

the entry of new firms. An increasing capital stock yields a process of successive rightward adjustment of the short-run marginal cost curve and, hence, of equilibrium output from X_s towards X_1 in Figure 1:2. Eventually, the industry is assumed to approach the new long-run equilibrium position, $(X_1, P_{X,1})$, determined by the new demand curve and the long-run marginal cost of output when all inputs are variable, $LRMC(X)$. This equilibrium corresponds to the long-run equilibrium in classical capital theory.¹⁾

The results in the Marshall-Viner model arise in our model as a special case, namely, the case of an unexpected permanent demand shift. The most important extensions in the current study as compared to the Marshall-Viner model are:

1. We extend the analysis of the Marshall-Viner case with an unexpected permanent shift by considering both

1) In addition Marshall analyzes the very short-run, where supply is represented by a vertical line, the intersection of which with the new demand curve determines output price.

the determination of prices and quantities for output and inputs in the period between short- and long-run equilibria and the determinants of the associated adjustment speed.

2. The current theory of adjustment is based on the solid microeconomic model of a present value maximizing firm. The Marshall-Viner model simply assumes that firms expand capacity at a high output price, but does not deal with the question of how firms determine their optimal capacity expansions.

3. The current theory, unlike the Marshall-Viner model, also determines the firm's and industry's adjustment to expected permanent shifts and to different types of temporary shifts.

1.5.3 The Accelerator and Partial Adjustment Models

The simple accelerator model, developed by Bickerdike (1914) and Clark (1917), states that net investment is proportional to changes in sales or demand, approximately measured by change in output:

$$(1.1) \quad I_{\text{net}} = b (X - X_{-1}),$$

where I_{net} is net investment, b is the desired quotient between capital and output and X is output. This model predicts that capital adjustment takes place within one time period. Empirical estimations have tended to produce a lower value for b than the average capital-output ratio.¹⁾ These results led Goodwin (1951), Chenery (1952), and Koyck (1954) to extend the accelerator model by allowing for partial capital adjustment as follows:

$$(1.2) \quad I_{\text{net}} = \eta (bX - K_{-1}),$$

where η is the share of the gap between desired and actual capital that is filled within the time period. This model implies that net investment is determined by current and past output changes. The weights for lagged output changes decline geometrically. Meyer and Kuh (1955), Jorgenson (1963), Smyth (1964) and Griliches

1) See for example Kuznets (1935).

(1960) and Griliches and Wallace (1965) examine less restrictive forms for the effects of lagged output changes. A general distributed lag form may be expressed as:

$$(1.3) \quad I_{\text{net}} = d_0 (X - X_{-1}) + d_1 (X_{-1} - X_{-2}) + \dots + d_n (X_{-n} - X_{-n-1}),$$

where d_i is the effect on investment from output changes lagged i periods. These authors arrive at the highly plausible conclusion that capital adjusts successively to changes in exogenous factors.

Jorgenson (1963) stresses that investment theory and investment functions in econometric studies should be based on the microeconomic theory of optimizing firms. Assuming exogenous and constant prices over time, Jorgenson determines desired capital as:

$$(1.4) \quad \text{Desired } K = \alpha \frac{P_X X}{C},$$

where α is output elasticity with respect to capital in Jorgenson's Cobb-Douglas production function and c is rental price of capital services. Net investment is determined by current and lagged changes in desired capital:

$$(1.5) \quad I_{\text{net}} = w(L) (\text{Desired } K - \text{Desired } K_{-1}),$$

where $w(L)$ is a power series in the lag operator.

Eisner and Strotz (1963) integrates different types of adjustment costs into a microeconomic model. These adjustment costs result in increasing marginal cost of investment and, thus, successive capital adjustment. The firm's optimizing behavior produce the simple partial adjustment model:

$$(1.6) \quad I_{\text{net}} = \eta (\text{Desired } K - K),$$

where Desired K is the long-run equilibrium capital at current prices. Moreover, the share of adjustment, η , is an endogenous variable determined as part of the firm's optimization.

The possibility of interdependent adjustment costs for several inputs have led Lucas (1967a), Nadiri and Rosen (1969), Coen and Hickman (1970), Helliwell and Glorieux, (1970), Schramm (1970), Nadiri (1972) and Kanis (1979) to develop and estimate models of type (1.6), where I_{net} is a vector of flows of inputs, K is a vector of stocks of inputs, while η is a matrix of adjustment coefficients. The flow of, say, net hiring of labor is then a linear function of the differences between desired and current stocks of labor, capital and other inputs.

Following Jorgenson (1963) and Eisner-Strotz (1963) latter theoretical developments of partial adjustment models typically have a microeconomic basis. This type of investment theory tends to merge into the theory of the firm's investment decision which, is analyzed in the next section.

Partial adjustment models based on the firm's optimization describe investment as determined by exogenous prices for output, investment goods and other inputs. The assumption of exogenous prices is legitimate for the firm in a competitive market but is not defensible for the industry. Hall and Jorgenson (1967), for example, use a partial adjustment model to estimate the effects of tax changes on the industry's investment, assuming output and capital prices to be unaffected by these changes. These assumptions are satisfied only if demand for output and supply of investment goods are both perfectly elastic for the industry. The application of a model of the firm's investment decision to the behavior of the industry is, therefore, an inappropriate generalization from micro to macro behavior.

Under some simplifying assumptions our investment model for the industry reduces to the partial adjustment model (1.6), thereby providing a solid microeconomic basis. The frequent empirical estimations of the par-

tial adjustment model are typically made at this level of aggregation.¹⁾

Our model suggests the following extensions to earlier partial adjustment models:

1. Instead of treating prices or output as exogenous variables for the industry, all prices and quantities are endogenous variables.
2. Investment is determined for any expected paths of the exogenous factors instead of being restricted to stationary conditions as in Eisner-Strotz (1963) and Lucas (1967). Investment might then be interpreted as an optimal capital adjustment not only towards a constant target but also towards a moving target.
3. Desired capital is determined from weighted averages of future expected demand, supply and production functions. The weights determine the degree of forward-looking investment decisions. These weights are found to depend on the rate of interest, the rate of depreciation, the long-run growth rate of demand, the elasticity of the price of investment goods and the elasticity of the marginal revenue product of capital.
4. The partial adjustment model arises as a special case of our investment theory under certain quite restrictive assumptions. This is analyzed in Chapter 6. Our model is not restricted to the cases where these assumptions are met. Instead, we analyze circumstances where the partial adjustment model might be a reasonable approximation, and the modifications that are necessary when these conditions fail to be satisfied. These results have important implications for the empirical estimation of partial adjustment models.

1) See for example Nadiri and Rosen (1969), Coen and Hickman (1970), Helliwell and Glorieux (1970), Schramm (1970), Nadiri (1972) and Kanis (1979).

1.5.4 Adjustment and Investment Determination at the Firm Level

In traditional microeconomic theory the single firm faces exogenous input and output prices in a competitive market. The firm adjusts instantaneously to its short- and long-run equilibrium position determined by these prices. This is analyzed in 2.7 below.¹⁾ A shift in an exogenous factor, say, output price causes an instantaneous shift in capital stock yielding infinite investment in a model with continuous time. The firm's net investment is determined by current changes in exogenous factors only. In Jorgenson's (1973) terminology, the firm's dynamic optimization problem might be decomposed into myopic decisions at each point of time, i.e. the optimal decision at a certain point of time is determined by data from that point of time only. This contrasts sharply with the common view, supported by empirical analyses, that investment decisions are forward-looking.

These unappealing implications of instantaneous capital adjustment and myopic investment decisions have induced different approaches to reformulating the investment theory of the firm. Arrow (1968), Kamien and Schwartz (1974), and Nickell (1974, 1975, 1978) avoid instantaneous downward capital adjustment by constraining gross investment to be positive. This type of model, however, still predicts instantaneous upward capital adjustment and myopic decision rules.

A different solution suggested by Jorgenson (1967), restricts the analysis to cases with gradual changes in output price, wage rate and implicit price of capital services. Investment is then finite and capital adjusts gradually. As shown later, our model yields analogous results for the firm's optimal adjustment. An important difference is, however, that our analysis in-

1) C.f. Haavelmo (1960), Hirshleifer (1970), or Nickell (1978).

cludes a theory of the determination of price paths while these paths are taken as given by Jorgenson.

The dominating part of the literature avoids instantaneous capital adjustment by introducing internal adjustment cost at the firm level. This gives rise to increasing marginal cost of investment.¹⁾ From given expected price paths of inputs and output, the firm calculates the discounted sum of future marginal revenue product for a marginal investment. The investment level is selected where the marginal cost of investment is equal to this discounted sum.²⁾

These models imply finite investment and successive adjustment of the firm's capital to changes in input and output prices. Moreover, the firm's investment is forward-looking in the sense that current investment depends on expected future paths for the exogenous prices, as analyzed by Gould (1968).

The assumption of increasing marginal cost of investment for the firm is criticized by Rothschild (1971). He argues that the marginal cost is as likely to be constant or decreasing. Nickell (1978) also rejects the hypothesis of adjustment costs at the firm level as an explanation for successive adjustment, suggesting that it might instead be explained by a combination of uncertainty, delivery lags, irreversible investment and imperfect capital markets.

In all these models, the firm's optimal adjustment is determined from given expected paths for exogenous

1) See for example Eisner and Strotz (1963), Lucas (1967a), Gould (1968), Treadway (1969, 1970, 1971, 1974), Maccini (1973), Mortenson (1973), and the surveys by Söderström (1974, 1976).

2) If the firm's production function is homogeneous of degree one, the marginal revenue product of capital is independent of the firm's capital, implying that current investment is independent of future investment. For the general case with any given production function, current investment must be determined as part of an investment plan for the entire future. This is analyzed by Treadway (1969).

prices. These models do not include any theory of the determination of these price paths. To answer the question posed above of how a firm and an industry react to shifts in demand these models must assume the price paths during the adjustment process. Only then can one determine adjustment for the firm and the industry. It is clear that these models require to be supplemented by a theory determining the price paths during the adjustment process.

Such a theory is provided by our model on the solid microeconomic basis of maximizing firms with rational price expectations. The firm's investment is determined in a two-step procedure. The first step involves the determination of expected price paths as the equilibrium price paths for the industry. In the second step, the single firm uses these exogenous price paths to determine its optimal quantity plans in accordance with traditional neoclassical investment theory without internal adjustment costs for the firm. The industry's investment is determined as an aggregate of the firms' optimal investments. Our model implies infinite investment for the industry as a result of an increasing supply price for investment goods at the industry level. This represents a departure from previous models in that finite investment is implied from conditions for industry equilibrium.

Given this mechanism ensuring finite investment at the industry level, the questionable assumption of an increasing marginal cost of investment at the firm level is avoided. Investment might instead be infinite at the firm level. One firm might sell, say, its stock of trucks to another firm at a certain point of time. Such transactions are excluded in models which assume increasing marginal costs of investment for the firm.

The major extension in our theory of the firm's investment is the provision of a theory determining expected price paths as equilibrium price paths at the industry level, whereas expected price paths are taken as given in earlier investment theories. The traditional model

with exogenous price paths might, of course, be used to analyze changes affecting only a single firm. Applied investment theory, however, typically considers changes that affect all firms in an industry. The assumption of exogenous price paths then eliminates most of the interesting adjustment mechanisms.¹⁾ To analyze the effects of changes in demand for output, supplies of inputs, technology, interest rate and taxes a theory is needed where prices change as part of the adjustment process. This study intends to contribute to the development of such a theory.

1.5.5 Adjustment and Investment Determination at the Industry Level

The concept of an increasing supply curve for investment goods is used by Lerner (1944) and Clower (1954b) to explain successive capital adjustment at the industry level. More formal models with an explicit microeconomic basis of optimizing firms are analyzed by Arvidsson (1960) and Wijkman (1965) under the assumption that firms face exogenous price paths for output and non-durable inputs. Investment determination in these models is analogous to that where the marginal cost of investment increases as a result of internal adjustment costs for the firm, as analyzed in 1.5.4. Again, the models yield no answer as to how the price paths for output and nondurable inputs are determined.

Smith (1966) and Lucas (1967b) do, however, analyze adjustment at the firm and industry levels when output

1) This deficiency is noted by Gould (1968, p 52) in a paper where he himself assumes exogenous price paths: "...if the firm being considered is a typical one in its industry and if the wage increase is industry-wide, then it would seem more reasonable, economically speaking, to recognize that the increase in wages will increase costs for every firm. In most cases this will cause an industry-wide reduction in output and consequently an increase in output price... We now examine the effect of various parameter changes keeping in mind that the fixed price assumption limits somewhat the inferential value of the results."

price and, in Smith's work, also the price of labor are treated as endogenous variables which change as part of the adjustment process. Smith's formal analysis is confined to determination of long-run equilibrium positions. No explicit analysis is made of the determination of expectations and investment during the adjustment period.

Lucas, however, provides an explicit analysis of the adjustment process under the assumption of stationary price expectations. Expected prices are then equal to current prices. In the terminology of Hicks (1939), firms have unit elasticity of price expectations.

We now analyze the adjustment to the demand shift in Figure 1:3 below under Lucas' assumptions of stationary price expectations. As before, $SRMC(X)$ shows the industry's marginal cost for constant capital. The $LRMC(X)$ shows the marginal cost when capital is fully adjusted also. The prices of investment goods and labor are exogenous variables. This requires the highly restrictive assumption of perfectly elastic supplies of investment goods and labor for this industry. The firm's marginal cost of investment is the sum of the exogenous price of investment goods and the marginal internal adjustment cost. The latter increases with the firm's gross investment. This means increasing long-run marginal costs for the firm and the industry, as illustrated in Figure 1:3.

The assumption of stationary price expectations implies that Lucas' model is applicable for unexpected shifts only. The market is assumed to be in an original long-run equilibrium, $(X_0, P_{X,0})$, up to the time of the shift. The shift in demand results in the short-run equilibrium price $P_{X,s}$ in Figure 1:3. Since all prices are expected to remain constant over time, the single firm uses current output price, $P_{X,s}$, to calculate the sum of the discounted marginal revenue products from a marginal investment. Combined with the firm's marginal cost of investment, this determines optimal investment. Stationary price expectations and a production function

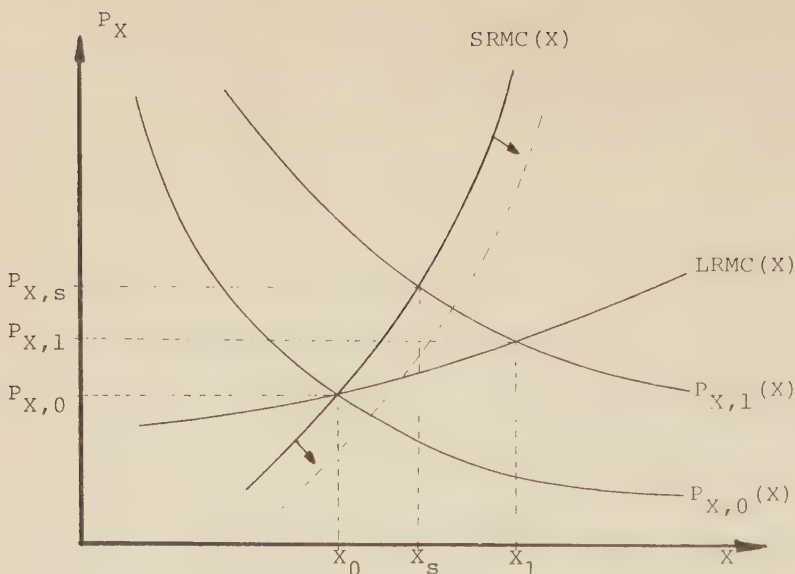


Figure 1:3 Adjustment process with stationary expectations.

that is homogeneous of degree zero imply that the firm plans constant investment over time. The firm, therefore, expects a constant marginal cost of investment and a zero capital gain.

Since the original long-run equilibrium price, $P_{X,0}$, implies zero net investment at the firm and the industry levels, the higher output price causes positive net investment. As capital increases, the short-run marginal cost curve shifts to the right, as illustrated in Figure 1:3. This results in an increasing output and a decreasing output price. The new long-run equilibrium position, $(X_1, P_{X,1})$, is reached asymptotically. Output price then yields gross investment equal to depreciation on the larger capital stock, i.e. net investment is zero.

As compared to the Marshall-Viner model Lucas extends the analysis through an explicit study of how a single firm and the industry adjust from the short- to the long-run equilibrium position.

The main deficiency with Lucas' solution is the assump-

tion of stationary expectations. This implies that firms make systematic mistakes during the adjustment process. At each point of time the firm expects output price to remain constant taking no account of the systematic price decreases which occur during the adjustment process. Investment plans based on stationary price expectations do not accomodate the systematic downward revisions arising from price changes at each point of time during the adjustment process. The firm does not recognize, therefore, the systematic capital losses resulting from the successive decreases in investment. The assumption of stationary expectations leads to the implausible conclusion that investors never learn from their own mistakes. Normally, we would not expect such investors to survive in a competitive market. Instead, more forward-looking or more receptive firms or firm managers would be able to make systematic gains at the expense of those with stationary expectations.

Our theory of adjustment assumes instead rational price expectations.¹⁾ Firms then make no systematic mistakes during the adjustment process. Investors realize that positive net investment leads to increases in capital stock and decreases in output price. The decreasing output price means, in turn, decreasing investment and expected capital losses during the adjustment process. This is analyzed further in the next chapter.

If Lucas' model is used to analyze temporary demand shift, its predictions are the same for the period before and during the shift as for a permanent shift. Due to the assumption of stationary expectations, Lucas' model does not distinguish between permanent and temporary shifts. This distinction appears, however, to be important both intuitively and empirically.²⁾ In contrast, our model predicts that the degree of capital

1) Cf. Muth (1961) and the survey by Kantor (1979).

3) Kanis (1979).

adjustment increases with the length of the time period of the temporary shift.

The most important extensions of our model as compared to that of Lucas are:

1. Our model determines the firm's and the industry's adjustments to unexpected changes under the assumption of rational price expectations. Lucas' model with stationary price expectations arises as a simple special case in our model. This is analyzed further in Section 3.8 below.
2. Unlike Lucas, prices of investment goods and labor are endogenous for the industry in our model.
3. The determinants of the speed of adjustment are examined in this study, while no such analysis is included in Lucas' model.
4. Due to the assumption of stationary expectations, Lucas' model is applicable to unexpected shifts only. Our model analyzes the effects of shifts with any information lead.
5. Finally, our model distinguishes between permanent and temporary shifts. No such distinction is made in Lucas' model.

1.5.6 Adjustment and Investment Determination for the Entire Economy

Ramsey (1928), Tinbergen (1956), Friedman (1962 ch. 13), Uzawa (1969) and Arrow-Kurz (1970) analyze the determination of time paths for aggregate saving and capital for an entire economy. Their assumption of decreasing marginal utility of consumption implies a relatively smooth consumption path and, hence, a relatively smooth path for aggregate investment. The assumption of decreasing marginal utility of consumption corresponds to our assumption of an increasing supply price of investment goods for the industry. Both assumptions imply an increasing marginal cost of investment and, thus, successive capital adjustment.

The dynamic optimization model developed by Arrow-Kurz (1970) is similar to our model for the special case where an unexpected permanent shift is introduced into an original stationary state (Section 3.3). Both determine optimal adjustment paths for capital towards a stationary state equilibrium, but they are used to study different questions. Arrow-Kurz analyze the determination of optimal aggregate saving path, whereas the current study analyzes market adjustment to different types of shifts in exogenous factors.

1.6 Summary of Chapter 2

We now outline the basic elements of our theory of adjustment which is presented in full form in Chapter 2. Consider a firm in a competitive market which receives information on an expected upward shift in demand one year ahead. How does the firm determine its optimal policy given this new information?

In the absence of future markets the firm must consider the development of the market, including the behavior of other agents. The firm is assumed to have a picture of how the market works, knowing that other firms also react to the future demand shift. The firm knows that other firms may increase investment before and after the demand shift, so it appreciates that the capital stock may increase and output price may decrease before the expected shift. Moreover, the firm may expect an upward shift in output price at the time of the expected demand shift, and a decreasing output price thereafter as a result of increasing capital stocks in other firms.

The firm must form expectations on future development of output and input prices that are reasonable with respect to other firms plausible reactions to the expected demand shift. We assume that the firm has a picture of the market in the form of expectations on the time paths of a simple demand curve for output, simple supply curves for inputs and a simple aggregate produc-

tion function for the industry. To form reasonable expectations on the time paths of input and output prices, the firm is assumed to act as if it calculates the equilibrium price paths for the market's inputs and output. Formally, the firm determines equilibrium price paths by solving an optimization problem at the industry level. Using these equilibrium price paths as expected price paths, the firm determines its optimal ex ante quantity paths. As long as no new information arises, equilibrium prices follow expected price paths and the firms follow their quantity plans.

Price expectations are, therefore, in accordance with the hypothesis of rational expectations. This hypothesis states that expectations should be set equal to the predictions of the relevant economic model.¹⁾

1.7 Summary of Chapters 3-5

The theory for the firm and the industry developed in Chapter 2 is used to examine adjustment to unexpected and expected permanent and temporary shifts in exogenous factors in Chapters 3-5. As an illustration the following conclusions arise in Chapter 3 for the special case with a permanent upward shift in demand.

1. A permanent shift in demand implies successive capital adjustment from the original towards the new long-run equilibrium position. This position is reached asymptotically. The results are consistent with long-run equilibrium in classical capital theory.
2. Information on a future demand shift causes an upward shift in investment and a windfall capital gain to capital owners. Investment then increases monotonically, thereby yielding an expected capital gain. A peak is reached at the time of the demand shift. Thereafter investment decrease monotonically towards the new long-run equilibrium level. This causes an expected capital loss during this part of the adjustment process. For the special case with an unexpected shift, investment decreases monotonically during the entire adjustment process. Rational price expectations give lower investment and slower capital adjustment as compared to 'stationary price expectations.

1) Cf. Muth (1961).

3. Capital adjustment before an expected shift causes output adjustment towards its new long-run equilibrium level. Output price adjusts in the direction opposite to the long-run effect. At the time of the demand shift, output and output price shift. The latter overshoots its new long-run equilibrium level. Thereafter output, output price and labor adjust monotonically towards their new long-run equilibrium positions, which are reached asymptotically.
4. A longer information lead gives rise to a smaller shift in investment and a lower windfall capital gain. Moreover, capital stock is higher at each point of time, implying higher output and lower output price. A longer information lead has, therefore, negative effects for capital owners and positive effects for purchasers of output.
5. The speed of capital adjustment increases directly in the depreciation rate, the long-run growth rate of demand, the interest rate, the absolute elasticity of the marginal revenue product of capital with respect to capital, and inversely in the elasticity of the supply price of investment goods. Numerical studies show that the depreciation rate is the most important determinant of the adjustment speed for reasonable ranges of the determinants.

The adjustment to permanent and temporary shifts in other exogenous factors, namely the supply curves, the industry level production function and the interest rate are analyzed in Chapters 4 and 5. In each case the conclusions are analogous to those for the demand shift. An expected shift in the supply curve for investment goods yields the different but intuitively appealing result that capital initially adjusts in the opposite direction from the long-run effect. This concludes the analysis of adjustment to basic shifts in exogenous factors.

1.8 Summary of Chapters 6 to 8

While Chapters 3-5 are confined to simple, mostly constant, paths for the exogenous factors, Chapter 6 examines investment determination for any expected time paths for these factors. Comparative dynamics results are derived for the effects on industry investment of any type of upward shift and for original paths for the exogenous factors. It is concluded that investment in-

creases directly with any type of upward shift in (1) current and future demand curves, (2) current and future supply curves of labor if the substitution effect dominates, (3) current and future marginal productivity functions for capital for a sufficiently high absolute price elasticity of demand and (4) future supply curves for investment goods. Investment decreases in (1) the size of the existing capital stock, (2) the interest rate, (3) the current supply curve for investment goods and (4) current and future marginal productivity functions for labor if the substitution effect dominates. Most of the results for basic shifts in Chapters 3-5 carry over to the case with any type of shift and any original expected paths for the exogenous factors.

The determination of the degree of forward-looking in investment behavior is also analyzed in Chapter 6. This leads to the conclusions that (1) a higher depreciation rate, (2) a higher long-run growth of demand, (3) a higher interest rate, (4) a higher absolute elasticity of the marginal revenue product of capital with respect to capital, and (5) a lower elasticity of the supply price of investment goods increase the relative importance of the near future and, thereby, induce less forward-looking investment.

Chapter 6 also shows that our investment theory for the industry might be interpreted as an extension of Keynes' (1936) investment theory.

Chapter 7 shows that our industry model might be reinterpreted as a theory for a profit maximizing monopolist. More generally, the model might be reinterpreted as a normative theory for a social welfare maximizer, using different weights for benefits and costs that arise for different groups in the economy. The theories of a competitive market and a profit maximizing monopolist arise as special cases of this normative model. Interestingly, our model predicts the same type of adjustment and investment theories for a competitive market, a profit maximizing monopolist and a

hypothetical social welfare maximizer.

Chapter 8 shows that our investment theory predicts substantial investment fluctuations as a result of optimal capital adjustment at and after the emergence of new information on the development of the exogenous factors. Since new information arises frequently in a world with uncertainty, observed investment fluctuations might indeed be necessary for optimal capital adjustment. In contrast, earlier explanations of investment fluctuations are typically based on the notion of short-sighted investors.¹⁾

Finally, Chapter 8 examines the determinants of the degree of investment fluctuations. Fluctuations in demand for investment goods are found to be inversely related to the sum of the depreciation rate and the long-run growth rate of demand and to the absolute elasticity of the marginal revenue product of capital. A high interest rate also contributes to these fluctuations but is relatively less important. The elasticity of the supply price of investment goods determines whether the effects of a given change in demand are felt principally on volume or price.

1) See for example Keynes (1936), Hicks (1950), Goodwin (1951) and Åberg (1968).

2. OPTIMIZATIONS AT THE FIRM AND INDUSTRY LEVELS

2.1 Introduction

This chapter formulates models of the single firm's optimization and the implied behavior of the industry for any expected paths for the exogenous factors.

2.2 The Firm's Optimization Problem

2.2.1 Optimization for Given Price Paths

A competitive market is by definition a market where each agent considers all prices as exogenous with respect to his own actions. Given point expectations for future price paths for output, investment goods and labor, from the planning point of time, t_p :

$\{P_X(t), P_I(t), P_L(t) \text{ for } t \geq t_p\}$,

the single firm's optimization problem is:¹⁾

$$(2.1) \quad \max_{\{X^i, I^i, L^i\}} \int_{t_p}^{\infty} (P_X(t)X^i(t) - P_I(t)I^i(t) - P_L(t)L^i(t)) e^{-R(t, t_p)} dt$$

subject to:

$$F^i(K^i(t), L^i(t), t) - X^i(t) \geq 0$$

$$\dot{K}^i(t) = I^i(t) - \delta K^i(t)$$

$$(K^i(t), L^i(t), X^i(t)) \geq 0$$

$$K^i(t_p) \text{ given.}$$

Letting $\triangle$ denote "is defined as", the notation is as follows:

- $I^i(t)$ ~~gross investment~~ gross investment or flow of durable input for firm i
- $L^i(t)$ labor or flow of nondurable input
- $X^i(t)$ output
- $K^i(t)$ capital or stock of durable input

1) After minor reinterpretation, this formulation also covers the case with jumps in the capital stock. Cf. Vind (1967) and Arrow-Kurz (1970).

$\dot{K}^i(t) \triangleq \frac{\Delta K^i(t)}{\Delta t}$ net investment

$K^i(t_p)$ inherited capital at the planning point of time

δ rate of capital depreciation

$F^i(K^i, L^i, t)$ production function

$R(t, t_p) \triangleq \int_{t_p}^t r(s) ds$ discount factor for discounting cash flow at time t to time t_p .
 $r(s)$ is the instantaneous discount rate at time s . Note that
 $\frac{\partial R(t, t_p)}{\partial t} = r(t)$.

2.2.2 Determination of Price Expectations

Given the existence of complete future markets, the firm observes current and future prices for inputs and output. The firm then simply determines its optimal quantity paths as described in (2.1). The market system then fills the celebrated role of conveying information for decentralized decision-making.

In reality, a typical firm, however, faces a highly incomplete set of future markets. In order to determine its optimal quantity plan, the firm must instead form expectations on the future price paths for its inputs and output. We now turn to the question of how actual firms form these price expectations.

Since information is costly to acquire, the typical firm bases its price expectations on an explicit or implicit simplified model of the economy requiring a relatively small set of information. The typical firm does not, therefore, take into account all potential interactions between different markets that arise in general equilibrium models. Instead, the firm focuses on the development of the market in which it operates. Typically, the firm tries to get information on the development of demand and cost conditions from different sources. It might, for example, gather information from predictions and planning in the aggregate, from predictions for the market made by organizations such as branch institutes

and newspapers and from different types of market surveys. Using this information, firm managers form their views on the development of the market. This includes more or less explicitly stated expectations for future input and output prices.

On the basis of this picture of actual firms, the firm in our model is assumed to form price expectations in the following way. The firm is assumed to hold expectations in the form of a time path for a simple demand function, where demand at any point in time is determined by output price at that time only. This is represented by the simple inverted demand function expressing output price, P_X , as a function of aggregate output, $X(t)$, and time, t :

$$(2.2) \quad P_X(X(t), t).$$

The firm ignores possible cross-effects of, say, input prices on demand as well as possible cross-effects between different points of time.¹⁾

Similarly, the firm forms expectations on time paths for simple input supply functions, where the supply of an input at a given point in time is determined by its price at that time only. This is represented by the

-
- 1) In a world with uncertainty, where the firm realizes that new information might arise at future points of time, this demand function should be interpreted as the 'certainty equivalent' demand function. The demand function might then be interpreted as the expected demand function, $\sum_j P_X^j(X(t), t)$, where P_X^j is the Savagian (1954) subjective probability for event j and $P_X^j(X(t), t)$ is the demand function for this event. Adjustment should be made for the effects of different types of non-linearities such as risk aversion. Cf. Hartman (1972, 1973). Emergence of new information might affect the 'certainty equivalent' demand function through changes in (1) subjective probabilities for different events, (2) expected demand function for different events and (3) the adjustment for non-linearities. Passage of time will typically reveal the outcome of different types of random processes which cause adjustment in subjective probabilities and, thus, in $P_X(X(t), t)$.

inverted supply functions for investment goods and labor:

$$(2.3) \quad P_I(I(t), t)$$

$$P_L(L(t), t),$$

where $I(t)$ is aggregate investment and $L(t)$ is aggregate labor.

The firm's expectations on the production possibilities of all firms in the industry are summarized by an expected path for an industry level production function. This determines aggregate output at a given time as a function of aggregate capital and aggregate labor at that time only.

To obtain a simple relationship between the firms production functions and the industry level production function, capital is assumed to be transferable between firms without transaction costs. This assumption does not hold literally, of course, but provides necessary simplification and may be a reasonable approximation. If relative production functions and expectations are fairly stable across firms over time, this will generate relatively small capital transfers. Moreover, capital might be transferred between firms not only through physical transfers of equipment, but also without physical movements through, for example, sale or rent of part of a plant. Finally, expectations tend to be equalized among firms, since, for example, large desired capital reductions by other firms might be interpreted as a warning to an individual firm which plans to increase its capital stock in line with its own optimistic expectations.

The industry level or the macro production function is then:

$$(2.4)$$

$$F(K, L, t) \triangleq (\max_i \Sigma F^i(K^i, L^i, t) \mid \Sigma_i K^i = K, \Sigma_i L^i = L, K^i \geq 0, L^i \geq 0 \text{ for each } i)$$

The sums in (2.4) apply to a finite or an infinite number of existing and potential firms. Where capital is transacted without costs, maximum output occurs at the event of equilibrium in a competitive market (Prop. 2.1 below).

The single firm forms expectations on the macro production function and its own micro production function only. The firm does thus not necessarily know all other micro production functions.

The macro production function is assumed to be homogeneous of degree one. No such assumption is made for the micro production functions. Firms may have different production functions with increasing, constant and decreasing return to scale for different input ranges. The homogeneity assumption for the macro production function does, however, imply that each micro production function eventually has decreasing or constant returns to scale.¹⁾

Firms are assumed to have homogeneous expectations about the future development of the exogenous factors at the industry level. This assumption is meant as an approximation for an actual market with heterogeneous expectations, where the assumed homogeneous expectations correspond to some average expectation for existing and potential firms.

Finally, each firm is assumed to know that firms have homogeneous expectations and determine their, normally different, policies through the same procedure.

The firm's problem is then to formulate expectations for the price paths that are reasonable with respect to these demand and supply expectations and the beha-

1) Bentzel and Johansson (1959) show that identical micro production functions and equilibrium on a competitive market imply that the long-run macro production function is homogeneous of degree one. Since capital is transferred between firms without costs, this result arises in the short-run too in our model, providing support for the homogeneity assumption.

behavior of other firms. If the firm expects high future demand, it must consider the possible reaction from other firms in the form of increases in capital and output. High future demand is typically combined with high future aggregate capital stock, dampening the tendencies to high output prices. High aggregate investments tend to result in high prices of investment goods, thereby dampening the tendencies to capital increases. The firm must determine reasonable expected price paths with respect to all these interactions.

We next show that the firm might determine equilibrium price paths by maximizing present value for given price paths for the industry where prices are constrained to lie on the respective demand and supply curves. We assume that the firm forms price expectations as if it determines equilibrium price paths in this way. These expected price paths are then used to determine optimal quantity plans in the firm level optimization problem (2.1).

2.3 Equilibrium and Industry Level Optimization

2.3.1 Equilibrium

The above assumptions enable the firm to calculate equilibrium price paths from the planning point of time. These price paths are denoted

$$\{P_X^*(t), P_I^*(t), P_L^*(t) \text{ for } t \geq t_p\}$$

An equilibrium is defined as a set of time paths

$$\{P_X^*(t), P_I^*(t), P_L^*(t), X^{i,*}(t), I^{i,*}(t), L^{i,*}(t) \text{ for } i=1,2,\dots \text{ and } t \geq t_p\}$$

where:

- i. each firm maximizes present value for expected price paths which are also equilibrium price paths.

$$\begin{aligned}
 \text{ii. } P_X(\Sigma X^i, *, t) &= P_X^*(t) \\
 P_I(\Sigma I^i, *, t) &= P_I^*(t) \\
 P_L(\Sigma L^i, *, t) &= P_L^*(t)
 \end{aligned}$$

2.3.2 Formulation of the Industry Level Optimization Problem

The firm calculates the equilibrium price paths, given the constraint that these paths and the associated aggregate quantity paths, $\{X^*(t), I^*(t), L^*(t) \text{ for } t \geq t_p\}$, must lie on the demand curve for output and the supply curves for inputs, i.e.:

$$(2.5) \quad \left. \begin{aligned} P_X^*(t) &= P_X(X^*, t) \\ P_I^*(t) &= P_I(I^*, t) \\ P_L^*(t) &= P_L(L^*, t) \end{aligned} \right\} \quad \text{for each } t \geq t_p$$

Moreover, the firm knows that all firms maximize present value with respect to the equilibrium price paths. Since capital is transferable between firms without costs, these separate maximizations yield a maximum for the sum of the present values for given equilibrium price paths with respect to the industry level production function. An equilibrium hence solves the following industry level maximization problem:

$$(2.6) \quad \begin{aligned} &\text{maximize} \int_{t_p}^{\infty} (P_X^*(t)X(t) - P_I^*(t)I(t) - P_L^*(t)L(t)) e^{-R(t, t_p)} dt \\ &\{X, I, L\} \end{aligned}$$

subject to:

$$\begin{aligned}
 F(K, L, t) - X(t) &\geq 0 \\
 \dot{K}(t) &= I(t) - \delta K(t) \\
 (K(t), L(t), X(t)) &\geq 0
 \end{aligned}$$

$K(t_p)$ given.

Assumptions:

1. $P_X(X,t)$, $P_I(I,t)$, and $P_L(L,t)$ are assumed to be continuously differentiable with respect to their first arguments. $F(K,L,t)$, $F'_K(K,L,t) \triangleq \frac{\partial F(K,L,t)}{\partial K}$ and $F'_L(K,L,t) \triangleq \frac{\partial F(K,L,t)}{\partial L}$ are assumed to be continuously differentiable in K and L .
2. $0 < P_X(X,t) < \infty$ and $P'_X(X,t) \triangleq \frac{\partial P_X(X,t)}{\partial X} < 0$.
3. $P'_I(I,t) \triangleq \frac{\partial P_I(I,t)}{\partial I} > 0$. $P_I(I,t) < 0$ for some I and $\lim_{I \rightarrow \infty} P_I(I,t) = \infty$.
4. $P'_L(L,t) \triangleq \frac{\partial P_L(L,t)}{\partial L} > 0$.
5. $F(K,L,t)$ is homogeneous of degree one and strictly concave in K and L . Moreover, $F(0,0)=0$, $F'_K > 0$, $F'_L > 0$, $F''_{KK} < 0$, $F''_{LL} < 0$, $\lim_{K \rightarrow 0} F'_K = \infty$, $\lim_{K \rightarrow \infty} F'_K = 0$, $\lim_{L \rightarrow 0} F'_L = \infty$, $\lim_{L \rightarrow \infty} F'_L = 0$.
6. $r(t) > \varepsilon$ for some $\varepsilon > 0$.
7. $\delta > 0$.
8. $K(t_p) > 0$.

We then have the following proposition:

Proposition 2.1: An equilibrium is equivalent to a solution to (2.6) under restrictions (2.5).

Proof in Appendix p. 156.

The values for different variables in equilibrium are indicated by stars when necessary to avoid confusion.

2.3.3 Necessary and Sufficient Conditions

The assumptions for the demand function, the production function and the supply functions imply positive K , L , and X in equilibrium. The nonnegativity constraints for these variables are, therefore, ignored. The Hamiltonian is then:

$$H = (P_X^*(t)X(t) - P_I^*(t)I(t) - P_L^*(t)L(t)) e^{-R(t, t_p)} + \lambda_1(t) (F(K,L,t) - X(t)) + \lambda_2(t) (I(t) - \delta K(t)).$$

Inserting the equilibrium conditions (2.5) the necessary conditions for a maximum to (2.6) and hence for an equilibrium are:¹⁾

$$(2.7) \quad \dot{\lambda}_2(t) = -\frac{\partial H}{\partial K} = -\lambda_1(t) F'_K + \lambda_2(t) \delta$$

$$(2.8) \quad \frac{\partial H}{\partial I} = -P_I(I^*, t) e^{-R(t, t_p)} + \lambda_2(t) = 0$$

$$(2.9) \quad \frac{\partial H}{\partial L} = -P_L(L^*, t) e^{-R(t, t_p)} + \lambda_1(t) F'_L = 0$$

$$(2.10) \quad \frac{\partial H}{\partial X} = P_X(X^*, t) e^{-R(t, t_p)} - \lambda_1(t) = 0$$

$$(2.11) \quad \lambda_1(t) \geq 0, \quad \lambda_1(t) (F(K^*, L^*, t) - X^*(t)) = 0$$

The transversality conditions are:

$$(2.12) \quad \lim_{t \rightarrow \infty} \lambda_2(t) \geq 0, \quad \lim_{t \rightarrow \infty} \lambda_2(t) K^*(t) = 0$$

Since the Hamiltonian is concave in K , L , and I , conditions (2.7)-(2.12) are also sufficient conditions for a solution to (2.6) and hence for an equilibrium.¹⁾

Moreover, since $P_X(X, t)$ is strictly decreasing in X while $P_I(I, t)$ and $P_L(L, t)$ are strictly increasing in I and L respectively, the solution for the equilibrium price and quantity paths is unique (Proof in Lemma A.1 in Appendix p. 158).

To simplify the necessary and sufficient conditions, note that assumption 2, (2.9), (2.10) and (2.11) imply:

$$(2.13) \quad P_X(F(K, L, t), t) F'_L(K, L, t) - P_L(L, t) = 0.$$

The equation states that the marginal revenue product of labor is equal to the price of labor in an optimal solution. Since the left hand side is continuous and strictly decreasing in L , positive for a sufficiently low L and negative for a sufficiently high L , this condition determines a unique L for any given pair (K, t) . This rule is applicable for any capital, not only for equilibrium capital. By the implicit function theorem, assumptions 1, 2, 4, and 5 determine L as a continuously differentiable function of K , say, $G(K, t)$.²⁾ Using

1) Propositions 7 and 8 in Arrow-Kurz (1970), pp 48-49.

2) Hyttén-Cavallius and Sandgren (1964), p. 307.

this function, the marginal revenue product of capital might be written as a function of capital and time only:

$$(2.14) \quad V(K, t) \triangleq P_X(F(K, G(K, t)), t) F'_K(K, G(K, t)).$$

The following lemma states the appealing result that higher capital stock implies lower marginal revenue product of capital when labor is adjusted to satisfy (2.13), i.e.:

$$\text{Lemma 1: } V'(K, t) \triangleq \frac{\partial V(K, t)}{\partial K} < 0.$$

Proof in Appendix p. 159.

The necessary and sufficient conditions (2.7)-(2.12) then simplify to (Lemma A.2 in Appendix p. 159):

$$(2.15) \quad P_I(I^*, t) = \int_t^{\infty} V(K^*, s) e^{-\delta(s-t) - R(s, t)} ds$$

or

$$(2.15')$$

$$(r(t) + \delta) P_I(I^*, t) - P'_I(I^*, t) \dot{I}^*(t) - \frac{\partial P_I(I^*, t)}{\partial t} - V(K^*, t) = 0$$

$$(2.16)$$

$$\lim_{t \rightarrow \infty} P_I(I^*, t) e^{-R(t, t_p)} \geq 0, \quad \lim_{t \rightarrow \infty} P_I(I^*, t) e^{-R(t, t_p)} \dot{K}^*(t) = 0.$$

$$(2.17) \quad P_X(F(K^*, L^*), t) F'_L(K^*, L^*, t) - P_L(L^*, t) = 0$$

$$(2.18) \quad X^* = F(K^*, L^*, t).$$

2.3.4 Interpretations

Since a marginal investment at time t yields $e^{-\delta(s-t)}$ units of capital service at time s , (2.15) simply states that the price of investment goods is equal to the discounted sum of the future marginal revenue products from a marginal investment. To interpret (2.15') consider the costs that must be covered by an implicit or explicit price for capital services. This rental must clearly compensate for interest and depreciation calculated on the price of investment goods, $(r(t) + \delta) P_I(I^*, t)$.

Moreover, the rental must compensate for expected capital loss as determined by the decrease in the price of investment goods in equilibrium:

$$(2.19) \quad P_K^*(t) = (r(t) + \delta) P_I(I^*, t) - \frac{dP_I(I^*, t)}{dt} = \\ = (r(t) + \delta) P_I(I^*, t) - P_I'(I^*, t) \dot{I}^*(t) - \frac{\partial P_I}{\partial t}(I^*, t)$$

$P_K^*(t)$ is the implicit rental for capital services in equilibrium. Capital gains and losses in equilibrium is a part of equilibrium return on capital and is, therefore, an expected and not a windfall capital gain. Equation (2.15') simply states that this implicit price for capital services is equal to the marginal revenue product of capital in equilibrium.

Since (2.15), (2.15') and (2.16) are functions of K , I , and t only, this formulation allows for a recursive solution of the optimization problem. Equations (2.15) and (2.16) (or (2.15') and (2.16)) are first used to determine equilibrium paths for capital and investment. Given the equilibrium capital path, (2.17) and (2.18) determine the equilibrium paths for L and X in the second step in the determination of an equilibrium. This two step procedure highly simplifies the determination of the optimal solution.

2.3.5 Some Useful Lemmas

We next state two lemmas, which are needed in later analyses. If the first step in the solution to the optimization problem yields higher equilibrium capital stock in one case as compared to another (due, for example, to higher inherited capital stock) this implies, *ceteris paribus*, higher output in the second step in the determination of an equilibrium. More formally:

Lemma 2: If $K_1^*(t) > K_0^*(t)$ for a point of time where
 $P_{X,1}(X,t) = P_{X,0}(X,t)$, $P_{L,1}(L,t) = P_{L,0}(L,t)$, and
 $F_1(K,L,t) = F_0(K,L,t)$ for all X , K and L , then

- i) $X_1^*(t) > X_0^*(t)$
- ii) $L_1^*(t) \begin{matrix} < \\ = \\ > \end{matrix} L_0^*(t)$ if the sum of the elasticity of substitution and the price elasticity of demand is positive, zero, and negative respectively.

Proof in Appendix p. 160.

The assumptions also imply the following lemma:

Lemma 3: Investment is finite. Moreover, the planned investment path is continuous except possibly for points of time when $\frac{\partial P_I}{\partial t}(I,t)$ is infinite for some I .

Proof in Appendix p. 161.

2.3.6 Mechanisms to Reach an Equilibrium

We next consider briefly possible mechanisms that ensure an equilibrium in the absence of future markets and of a Walrasian auctioneer. The first condition for an equilibrium is satisfied by the assumptions for the firm's optimization process. If expected prices equal market clearing prices for the current point in time, current quantities in the solution to the industry level optimization problem satisfy the second condition for an equilibrium. If each firm has a unique optimal quantity plan, this must coincide with the solution obtained for the firm in the solution to the industry level optimization. The firms' optimizations at time t_p then ensure that the second condition is satisfied for the current point in time. A sequence of current markets then ensure that the second condition is satisfied as long as no new information arises.

If, however, some firms have multiple optimal quantity plans, some additional mechanism is needed to guarantee balancing quantities. We might imagine a tatonnement

process that ensures balancing current quantities through interaction among agents on current markets. If preliminary investment plans produce demand for investment goods in excess of that which will prevail at the equilibrium price, some firms would be induced to select another of their optimal investment levels through the lack of investment goods. Adjustments will continue until all current quantities balance. Firms with multiple optimal quantity paths at time t_p thus successively choose among these paths such that current aggregate quantities always balance. Again, the second condition is successively satisfied on a sequence of current markets.¹⁾

2.3.7 Market Behavior as Industry Level Optimization

In summary, we have the following theory for the firm and the industry. Firms have a picture of the market in the form of homogeneous expectations on the development of the market's output demand, input supplies, technology and interest. For any given expected time paths for these exogenous factors, the firm determines expected price paths as the equilibrium price paths by solving an industry level optimization problem.

Using these price expectations, the firm determines its quantity plans by solving the firm level optimization problem. Given that all firms determine their optimal policies in this way, and that some mechanism exists to balance current quantities an equilibrium arises.

Equilibrium prices on a sequence of current markets follow the expected price paths and each firm follows one of its optimal quantity plans as long as no new information arises. Price expectations are, therefore, consistent with the predictions of the relevant economic model, in accordance with the hypothesis of rational expectations. Rational price expectations do not, of course, imply perfect foresight. Instead this study focuses on the adjustment process that arises at the

1) Some mechanism ensuring balancing quantities is normally also needed in a competitive equilibrium model with a complete price vector, cf., Debreu (1959).

emergence of new information on current and future paths for the exogenous factors. The assumption of rational price expectations essentially states that firms make educated guesses of future price paths. This behavior of firms implies that the industry follows the price and quantity plans in the solution to the industry level optimization problem as long as no new information arises.

The determination of price and aggregate quantity paths might, therefore, be analyzed as the outcome of an optimization problem for the industry. This important result is used to analyze market adjustment and, more generally, determination of prices and aggregate quantity paths in the following chapters.

2.4 Ex Ante and Ex Post Paths

The solutions to the firm and the industry level optimization problems are optimal plans for expectations on exogenous factors held at a given point of time. The solid curve starting at time $t_{i,0}$ in Figure 2:1 illustrates the ex ante path for aggregate investment for given expectations at this point in time.

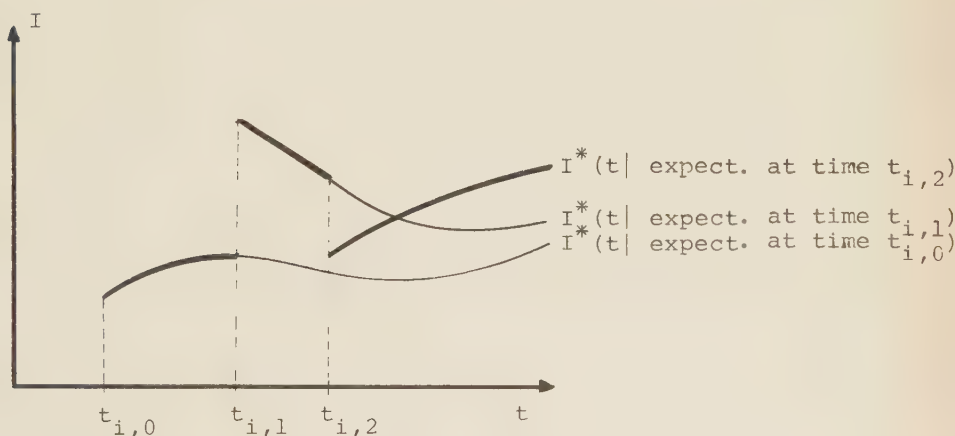


Figure 2:1. Ex ante and ex post investment paths.

As long as exogenous factors follow their expected paths price expectations are fulfilled and the ex post development follows the ex ante plans. If new information arises, the optimization problems are resolved for the new expected paths for the exogenous factors. This is illustrated by the new ex ante paths for aggregate investment starting at the information points of time $t_{i,1}$ and $t_{i,2}$ in Figure 2:1. The actual or ex post investment path is given by segments of the ex ante paths. This is illustrated by the thick curve in Figure 2:1.

Our theory of the firm assumes rational price expectations. The model allows, however, for any expected time paths and consequently any type of revision rules for expectations on exogenous factors. No specific assumption is made for the type of expectation formation for the development of output demand, input supplies, technology and interest rate.

2.5 Relation to General Equilibrium Theory

Our model is partial in the sense that exogenous time paths are assumed for a simple demand function for output and for simple functions for inputs. This represents a simplification as compared to the intertemporal general equilibrium model where demand and supply depend on all prices at all points of time.¹⁾ A shift in an exogenous factor in the general equilibrium model, such as preferences or technology, typically affects all prices and quantities at all points in time. The model does not, however, provide any answer to the question of how different prices and quantities are affected, and, therefore, sheds no light on the adjustment process. In order to obtain a meaningful and informative picture of market adjustment, simplifying assumptions are made in our model. As a byproduct of this simpli-

1) For the intertemporal interpretation of the general equilibrium model see Malinvaud (1953), Svensson (1976) or any standard textbook in general equilibrium theory as Malinvaud (1972).

fication, information required by firms to calculate equilibrium prices becomes similar to information available to and used by actual firms. By comparison the firm in the general equilibrium model requires an excessive set of information to calculate future equilibrium prices in the absence of future markets. This is discussed further in the next section.

2.6 Alternative Assumptions for Price Expectations

2.6.1 Price Expectations Using More Information

The Arrow-Debreu general equilibrium model determines prices and quantities from preferences and production possibilities for all goods at all points of time and all possible states of the world. In the absence of future markets the general equilibrium model might be interpreted as a model with temporary equilibrium on current markets and perfect foresight on future prices.¹⁾ This model suggests an alternative formulation of the firm's price expectations where a firm with rational price expectations bases its price expectations on information on preferences and production possibilities at all future points of time and all possible states of the world. Such a formulation would, however, require much more information than is available to and is used by a typical firm. Our model is instead based on the notion that the firm's information typically focuses on its own market. The firm determines its expected price paths by calculating equilibrium price paths for its market from simple output demand and input supply functions without considering all possible general equilibrium interactions with other markets. As a consequence of costly information, the

1) This temporary equilibrium model is developed by Hicks (1939) and Radner (1972). See also Svensson (1976). An equilibrium in our simplified model is a similar temporary equilibrium with price expectations equal to future equilibrium prices.

firm bases its price expectations and its behavior on a partial equilibrium model of the economy.

2.6.2 Price Expectations Using Less Information

Lucas' (1967b) model for firm and industry adjustment represents a step in the opposite direction by limiting the set of information to current prices through the assumption of stationary price expectations. This assigns, in my view, too narrow limits to the set of information used by a typical firm. Stationary price expectations imply that expectations of future changes in exogenous factors are irrelevant to the determination of investment and other quantities by the firm. This implication contrasts sharply with the fact that firms spend different types of resources on acquiring information on the future development of its market. Moreover, it contrasts with the common view among businessmen, administrators and applied economists that investment is a forward-looking activity. This is stressed, for example, by Keynes (1936).

The performance of a firm is highly dependent on the managers ability to form proper expectations on the basis of various sources of information. To be successful a firm needs managers with adequate knowledge and a good intuitive feeling for its market. In a more or less competitive setting for managers or firms surviving managers tend to be the most skillful in these respects.

As shown in Section 3.8 below, the assumption of stationary expectations implies that systematic profits might be made by taking account of predictable price changes. Entry of new more forward-looking firms and changes within existing firms might be expected to eliminate firms and firm managers with stationary expectations, the outcome being a market with more forward-looking expectations.

The current formulation, where the firm bases its price expectations on expected developments for simple exogenous factors avoids both the excessive information required in a general equilibrium model and the narrow limits set by the assumption of stationary expectations.

2.7 Exogenous Investment Price at the Industry Level

A horizontal supply curve for positive as well as negative quantities of investment goods implies that the price of investment goods is determined outside the market. This price is then exogenous to the industry, violating our assumption 3. This case is examined, nevertheless, to serve as a point of reference. In this case, (2.15') reduces to

$$(2.20) \quad V(K,t) = (r(t) + \delta)P_I(t) - \frac{dP_I(t)}{dt}$$

$P_I(t)$ is the exogenous price of investment goods. Equation (2.20) shows that capital stock is determined by current exogenous factors and the current change in the price of investment goods only. A shift in demand consequently causes an instantaneous capital adjustment and infinite investment for the industry.

3. ADJUSTMENT TO PERMANENT DEMAND SHIFTS

3.1 Introduction

This chapter analyzes market adjustment to different types of shifts in demand. We use the earlier result that the determination of prices and aggregate quantities might be analyzed as the outcome of an optimization process for the industry. A fairly rigorous analysis is made for the case with a demand shift. The effects of shifts in other exogenous factors are analyzed in Chapter 4 with analogous results.

An upward shift in demand is defined as a shift from an expected path for the demand curve, $\{P_{X,0}(X,t)\}$, to another path, $\{P_{X,1}(X,t)\}$, such that $P_{X,1}(X,t) \geq P_{X,0}(X,t)$ for each X and t with strict inequality for each X for some time period starting at the time of the shift, t_s .

Such shifts are illustrated for a constant output, $\bar{X}$, in Figure 3:1

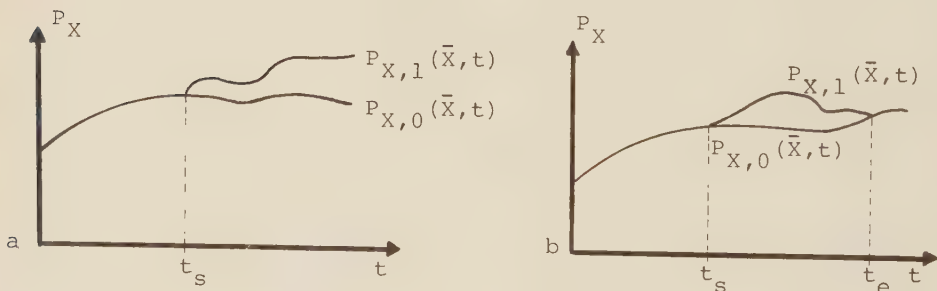


Figure 3:1. Upward shifts in demand.

A temporary upward shift in addition requires that $P_{X,1}(X,t) = P_{X,0}(X,t)$ for each X and t after some point of time. The temporary shift thus ends at time, t_e , as illustrated in Figure 3:1.b.

The major part of this chapter analyzes the simple case of a basic permanent demand shift as illustrated in Figure 3:2.

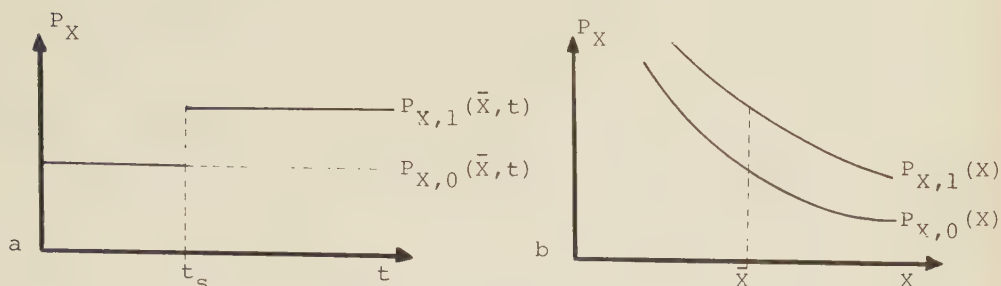


Figure 3:2. Basic permanent shift in demand.

A stationary state is defined as a state where all quantities and prices remain constant over time. Adopting the standard procedure of starting from an equilibrium, we begin the analysis from a stationary state. All exogenous factors are constant over time and the stationary state would continue indefinitely in the absence of a demand shift. At time t_1 the agents on the market receive information which anticipates a basic upward shift in demand at the future point of time t_s . We now examine the market's adjustment to this demand shift when all exogenous factors follow their at time t_1 expected paths and no additional information arises during the adjustment process. We are thus concerned with a comparative dynamics study of the effect of a basic permanent shift in demand.

We examine adjustment for three cases: (1) an unexpected shift, i.e. $t_s - t_1 = 0$, (2) an expected shift, i.e. $0 < t_s - t_1 < \infty$, and (3) the limiting case with perfect foresight, i.e. $t_s - t_1 \rightarrow \infty$.

3.2 Stationary States

3.2.1 Determination of the Stationary State Solution

From (2.6) and (2.15') the time paths for capital and

investment in a solution to the industry level optimization problem (2.6) must satisfy the following equations:

$$(3.1) \quad \dot{K}(t) = I(t) - \delta K(t)$$

$$(3.2) \quad \dot{I}(t) = \left[(r(t) + \delta) P_I(I, t) - \frac{\partial P}{\partial t} I(I, t) - V(K, t) \right] / P_{II}^I(I, t)$$

In order to get an original stationary state solution in the absence of any demand shift all exogenous factors are assumed to be constant over time. (3.1) and (3.2) then reduce to:

$$(3.3) \quad \dot{K}(t) = I(t) - \delta K(t)$$

$$(3.4) \quad \dot{I}(t) = \left[(r + \delta) P_I(I) - V(K) \right] / P_{II}^I(I)$$

The loci of points where capital respective investment are constant are shown in Figure 3:3. The station-

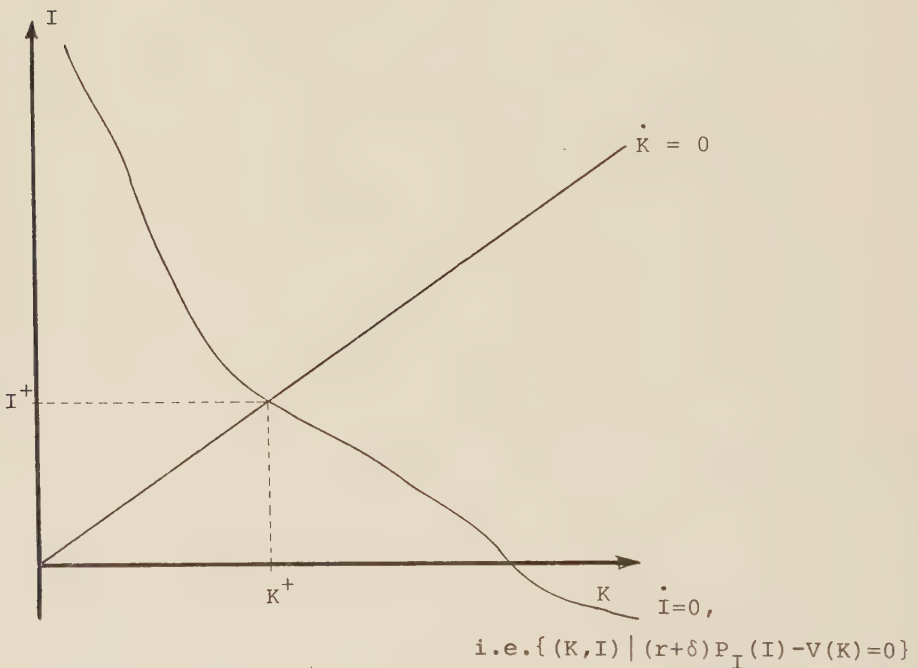


Figure 3:3. Determination of stationary state capital and investment.

nary state solution, is determined by the unique intersection of these curves, as illustrated in Figure 3:3. A stationary state solution is indicated by plus signs, (K^+, I^+) . In the second step stationary state capital determines stationary state solution for labor and output through Equations (2.17) and (2.18).

If all exogenous factors are constant and inherited capital is equal to K^+ , Equations (3.3) and (3.4) produce constant capital and investment when investment equals I^+ . This results in constant X , P_X and L . We have, therefore, a stationary state solution to the optimization problem (2.6).

Stationary state output and output price might alternatively be determined from the demand curve and the long-run marginal cost curve, as illustrated in Figure 3:4. The long-run marginal cost curve is defined as the marginal cost of output when capital and labor are selected to minimize total costs, i.e., when:

$$(3.5) \quad \frac{F'_K(K, L)}{F'_L(K, L)} = \frac{P_K(K)}{P_L(L)} = \frac{(r+\delta)P_I(I)}{P_L(L)}$$

Since $P_I(I)$ and $P_L(L)$ are strictly increasing and the production function is homogeneous of degree one the long-run marginal cost curve is strictly increasing in X , as illustrated in Figure 3:4.

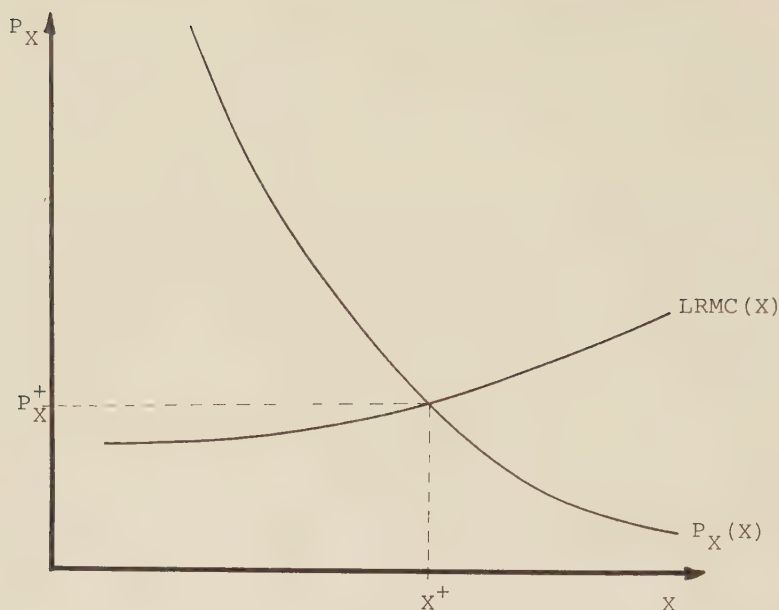


Figure 3:4. Determination of output and output price.

3.2.2 Comparative Statics of a Shift in Demand

By assumption we start from an original stationary state, $(K_0^+, I_0^+, X_0^+, P_{X,0}^+)$, that will continue indefinitely in the absence of a shift in demand. The demand shift disturbs this original long-run equilibrium and brings about a new stationary state. In the next section we demonstrate the important result that the adjustment process involves quantities and prices that converge to this new long-run equilibrium solution. In this section we examine the long-run effect of a shift in demand through a comparative statics study of long-run equilibrium solutions.

An upward shift in the demand function produces a higher marginal revenue product for each level of capital stock. This is represented by an upward shift in the $V(K)$ -function (Lemma A.3 in Appendix p. 161).

The demand shift, therefore, induces a rightward shift

in the $\dot{I}=0$ -curve resulting in higher stationary state capital and investment as illustrated in Figure 3:5.
(Proposition 3.1 provides a formal proof.)

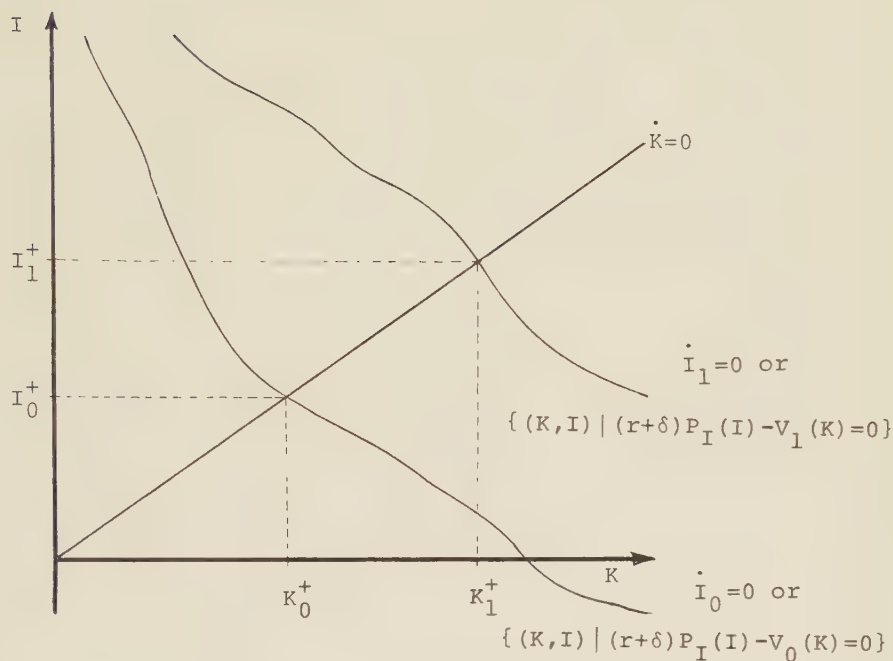


Figure 3:5. Comparative statics of the effects on capital and investment.

Moreover, due to the increasing long-run marginal cost curve, a higher demand yields higher long-run equilibrium output and output price, as illustrated in Figure 3:6.

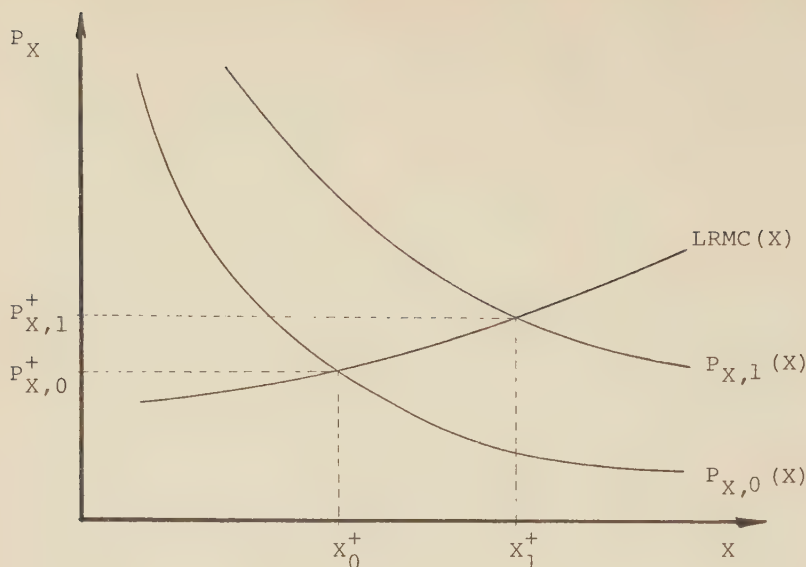


Figure 3:6. Comparative statics of the effects on output and output price.

Formally, we have the following results:

Proposition 3.1: (Comparative Statics): An upward shift in demand results in strictly higher stationary state capital, investment, output, output price and labor.

Proof in Appendix p. 163.

The effects of a demand shift on the long-run equilibrium are, therefore, in accordance with traditional comparative statics results for such a shift.¹⁾ Classical capital theory for a stationary economy as developed by Böhm-Bawerk (1891), Wicksell (1901, 1934) and Malinvaud (1953) also deals with these long-run equilibrium solutions. Our results are consistent with this classical capital theory in the sense that the long-run equilibrium in this theory coincides with long-run equilibrium or stationary state in our model.

The classical theory and the comparative statics analysis do not, however, answer the question of how the

1) See for example Smith (1966).

market adjusts from one long-run equilibrium position to another. Before turning directly to this issue, we show that the stationary state may be interpreted in terms of growth deflated variables, thereby allowing for long-run growth trends in basic variables. These results are later used to show that a change in the long-run growth rate has effects similar to those arising from a change in the depreciation rate.

3.2.3 Long-Run Growth and Technological Progress

If the market shows a long-run growth trend at rate g_1 for basic variables X_b , K_b and L_b this produces a stationary state for the growth deflated variables defined as:¹⁾

$$X(t) \triangleq X_b(t) e^{-g_1 t}, \quad K(t) \triangleq K_b(t) e^{-g_1 t}, \quad L(t) \triangleq L_b(t) e^{-g_1 t}$$

By construction of such growth deflated variables the adjustment on a market with a long-run growth trend might be analyzed as an adjustment between stationary states for growth deflated variables. To obtain the paths for basic variables, growth deflated variables are simply multiplied by the factor $e^{g_1 t}$, i.e.,

$$X_b^*(t) = X^*(t) e^{g_1 t}.$$

The basic variables K_b , I_b and L_b should be measured in efficiency units in order to yield a production function that tends to be constant over time. Embodied and disembodied technological progress introduce divergences between basic and physical units for capital, investment and labor.

The rate of depreciation in basic capital is denoted δ_b . Basic capital is then:

$$K_b(t) \triangleq \int_{-\infty}^t I_b(s) e^{-\delta_b(t-s)} ds.$$

The definition of growth deflated capital implies:

1) This type of growth deflating is applied by Arrow (1968) and Arrow-Kurz (1970).

$$(3.6) \quad \dot{K}(t) = \frac{d(K_b(t)e^{-g_1 t})}{dt} = I(t) - (\delta_b + g_1)K(t) = I(t) - \delta K(t)$$

To obtain a constant growth deflated capital, $\dot{K}=0$, investment must clearly compensate for depreciation in basic capital and long-run growth. The depreciation rate in the above model should be interpreted as the sum of the depreciation rate in basic capital and the long-run growth rate.

3.3 Adjustment to Unexpected Demand Shifts

3.3.1 Capital and Investment

We first examine adjustment to an unexpected shift in demand. The market is in an original stationary state up to the time of information, which is the time of the shift in this special case. At this point in time the firms determine new investment plans based on their new expectations. The development of prices and aggregate quantities may be described by the solution to the industry level optimization problem (2.6) for the new expectations with the planning point of time, t_p , equal to the information point of time, t_i . This adjustment path is followed as long as no new information arises. We now study the determination of the adjustment path at time t_i .

In the first step of the solution to the dynamic optimization problem time paths for capital and investment are determined. These are illustrated in the phase diagram of Figure 3:7. The original demand curve prevailing up to time t_s corresponds to the marginal revenue production function $V(K, t < t_s)$, the locus of points with constant investment, $\dot{I}(t < t_s) = 0$, and the original stationary state, (K_0^+, I_0^+) , in Figure 3:7. The upward demand shift causes an upward shift in the marginal revenue product function to $V(K, t \geq t_s)$ and a northeast shift in the locus of points with constant investment to $\dot{I}(t_s) = 0$ in Figure 3:7.

Equations (3.3) and (3.4) derived from the necessary

conditions for an optimal solution determine capital and investment changes for any point (K, I) in Figure 3:7. For points southeast of the $\dot{K}=0$ -line gross investment is insufficient to compensate for capital depreciation, with a resulting decrease in capital stock. This is illustrated by horizontal arrows in Figure 3:7. Analogously, capital increases for points northwest of this line.

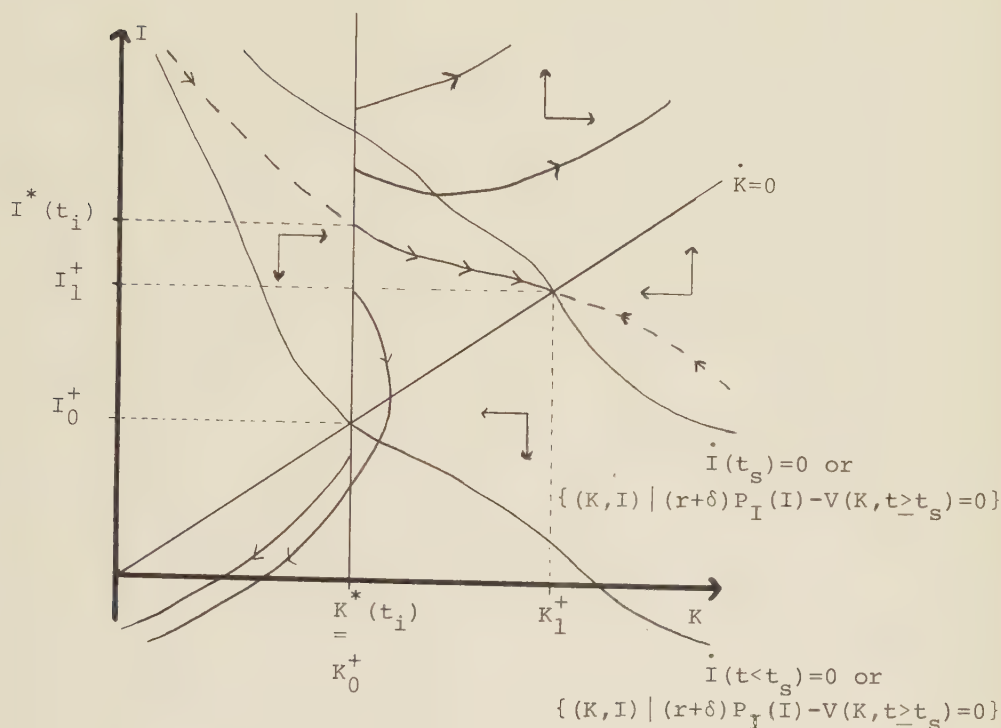


Figure 3:7. Adjustment for an unexpected demand shift.

For any point northeast of the $\dot{I}(t_s)=0$ -curve, (3.4) implies increasing investment. This important result is explained by noting that (3.4) corresponds to the necessary condition (2.15'). This equation states that expected capital gain, $P_I^* \dot{I}^*$, is equal to the difference between interest and depreciation costs associated with holding capital, $(r+\delta)P_I(I^*)$, and $V(K^*)$. For points

northeast of the $I=0$ -curve the relatively high investment involves high costs (since $P'_I > 0$) and the relatively high capital stock yields a low marginal revenue product of capital (since $V' < 0$). Consequently capital gain must be positive to fill the gap between high costs and low marginal revenue product. This requires increasing marginal cost of investment over time and thus increasing investment northeast of the $I=0$ -curve. Analogously, investment decreases southwest of this curve. This is illustrated by vertical arrows in Figure 3:7.

Since investment is finite the initial stock of capital is given in the original stationary state, i.e., $K^*(t_i) = K_0^+$, as shown in Figure 3:7. The necessary conditions (3.3) and (3.4) determine the entire future path for capital and investment for any given starting point, $(K(t_i), I(t_i))$. For high initial investment both capital and investment become infinite. For some intermediate investment the path converges to the new stationary state point. Both capital and investment become negative for a low initial investment. These paths are shown by arrowed curves in Figure 3:7.

It remains to determine optimal initial investment, $I^*(t_i)$. To do this we need the following proposition which states the important result that the adjustment path converges to the new long-run equilibrium position:

Proposition 3.2: If all exogenous factors are expected to be constant after some point of time, t_f , capital and investment converges to the stationary state position determined by these exogenous factors

Proof in Appendix p. 164.

Accordingly, the adjustment to the unexpected demand shift is given by the unique solid curve starting at $K^*(t_i)$ and converging to the new stationary state position in Figure 3:7. The time paths for capital and investment are also shown by the curves for a zero infor-

mation lead, $t_s - t_i = 0$ in Figure 3:9.b-c below.

The broken curve in Figure 3:7 shows all points where the path eventually converges to the stationary state point (K_1^+, I_1^+) . For any inherited capital, not necessarily determined by an original stationary state, the adjustment path is a part of the broken curve. (Prop. 3.2.) The starting point is determined by inherited capital and adjustment follows the broken curve towards the stationary state position.¹⁾

3.3.2 Adjustment Path for Output and Output Price

In the next step in the determination of the equilibrium solution the above results for the capital path are used to determine the adjustment paths for output and output price.

Since investments are finite, $(x^*(t_i), p_x^*(t_i))$ must lie on the short-run marginal cost curve. This curve shows the marginal cost of output as labor is varied for the

-
- 1) The broken curve in Figure 3:7, may cross the horizontal axis for a sufficiently large capital stock. Such a large inherited capital then causes negative gross investment for some time period. The assumptions of the above model do not exclude negative gross investment a priori. The industry might sell part of its capital stock to some other industry or for scrapping. The increasing supply curve for investment goods implies that the price of investment goods is relatively low for negative gross investment.

For an industry with specialized equipment, for example, the world market for shipping services, the supply curve for investment goods might be such that gross investment is always positive. This is the case if $P_I(0, t)$ is negative for all t . The assumptions of strictly positive output price and marginal product of capital imply that the right hand side in (2.15) is strictly positive. This results in strictly positive investment for such a supply curve. In terms of Figure 3:7 this means that the $\dot{I}=0$ -curve does not intersect the horizontal axis, implying that the broken curve produces positive gross investment for any inherited capital. Our model therefore includes as a special case circumstances where gross investment is constrained to be positive. This case is analyzed by Arrow (1968), Kamien-Schwartz (1974) and Nickell (1974, 1975, 1978).

given inherited capital. Starting at the original stationary state, $(X_0^+, P_{X,0}^+)$, the unexpected demand shift at time t_1 produces an instantaneous shift to $(X_s^*, P_{X,s}^*)$ in Figure 3:8.

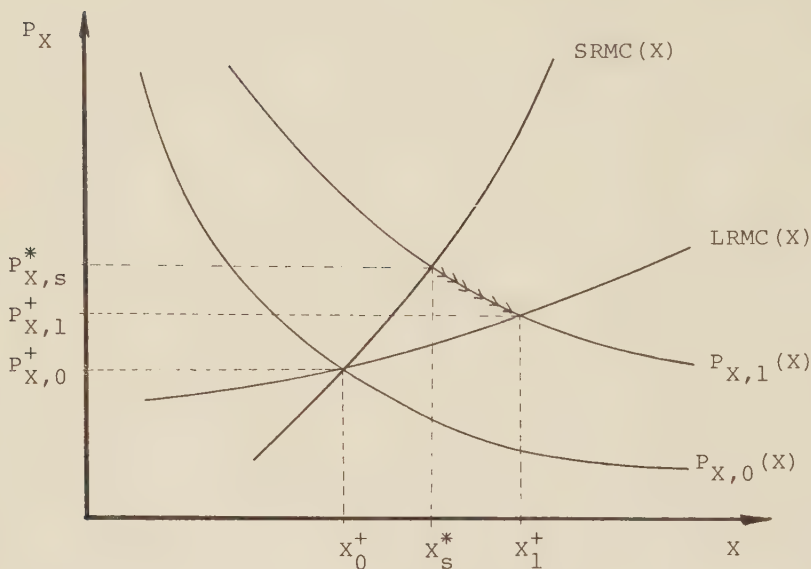


Figure 3:8. Adjustment for an unexpected demand shift.

As shown in Figure 3:7, capital increases towards its new stationary state value from the time the adjustment starts. Consequently, output increases towards its new stationary state value from time t_1 (Lemma 2 p. 45). The adjustment of output and output price is indicated by the path along the new demand curve marked by arrows in Figure 3:8. The paths are also shown by curves $t_s - t_1 = 0$ in Figure 3:9.d-e.

Our results for output and output price adjustment in response to an unexpected shift are consistent with traditional Marshall-Viner analysis applying short- and long-run marginal cost curves. Our results extend traditional results by providing a solid microeconomic basis for the adjustment path. This includes a determination of each firm's optimal investment path under

rational price expectations. We also determine the aggregate investment path, as shown in Figure 3:7. The short-run stickiness of capital arises as an implication of the analysis. Our model determines not only the instantaneous and the asymptotic effect of an unexpected shift but also the effect in the period between the short- and long-run equilibria. Section 3:9 studies the determinants of the adjustment speed between two long-run equilibrium positions. In addition, we later examine the adjustment to permanent and temporary demand shift with any information lead, whereas the Marshall-Viner analysis is confined to the special case with a permanent shift and a zero information lead.

3.3.3 Conclusions for an Unexpected Demand Shift

The following conclusions arise for an unexpected upward shift in demand:

1. Output and output price shift at time t_s . Output price overshoots as compared to its new long-run equilibrium level. Thereafter, capital, output and output price adjust successively towards their new stationary state positions. (Curves $t_s - t_i = 0$ in Figure 3:9.c-e.)
2. Investment shifts upwards at time t_i ($= t_s$) and overshoots as compared to the new long-run equilibrium position. As capital increases towards the new stationary state position investment decreases monotonically towards its new higher stationary state level. (Curve $t_s - t_i = 0$ in Figure 3:9.b.) Analogously, larger initial capital implies lower initial investment, as revealed by the broken curve in Figure 3:7.
3. The upward shift in investment at time t_i produces an unexpected upward shift in the price of investment goods, i.e., a windfall capital gain for capital owners. The monotonic decrease in investment during the adjustment yields expected capital losses

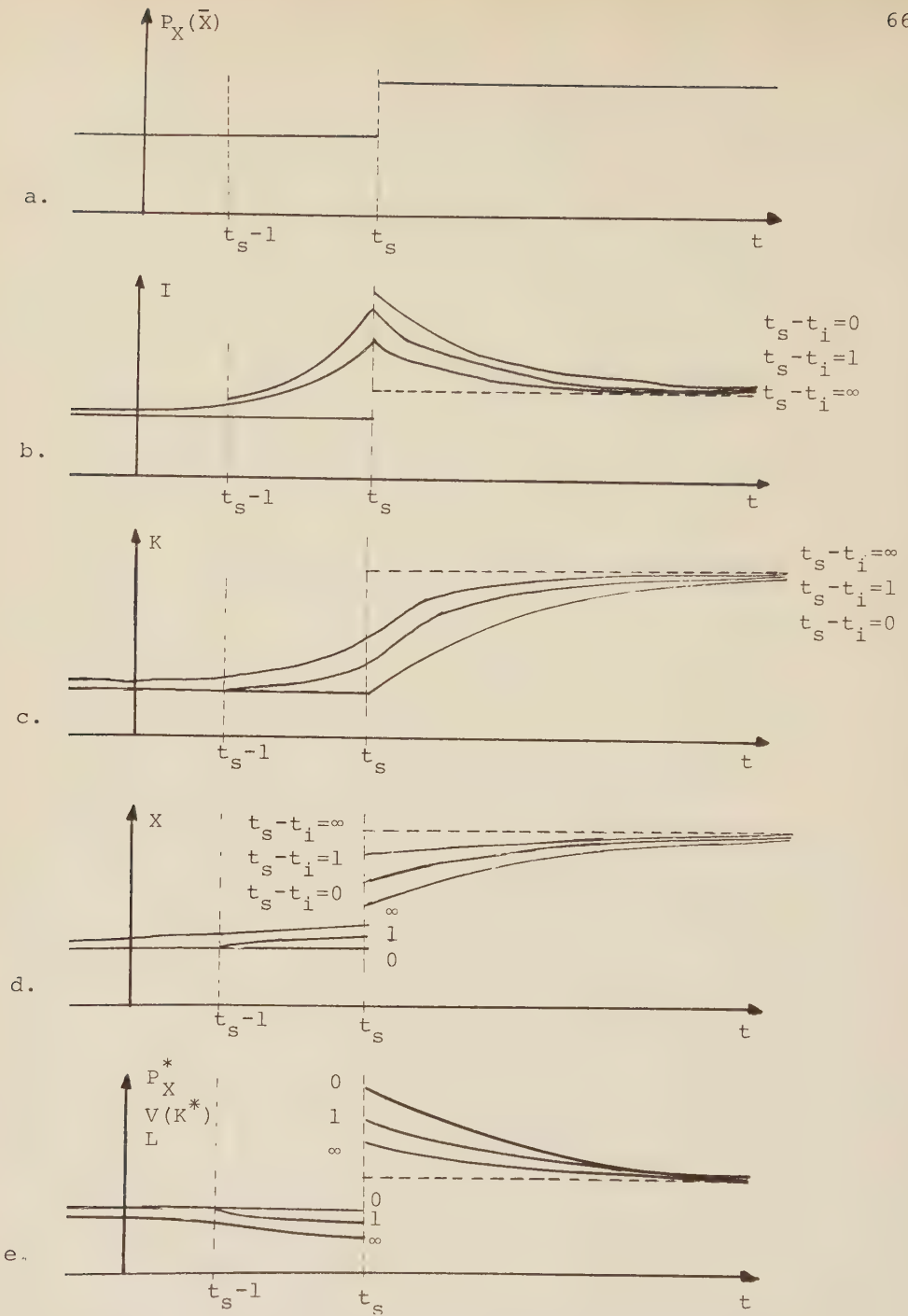


Figure 3:9. Adjustment for an upward shift in demand with different information leads. Broken lines show stationary state values. The substitution effect is assumed to dominate in Fig. e.

that fill the gap between the relatively high marginal revenue product of capital, $V(K^*)$, and the relatively low interest and depreciation costs, $(r+\delta)P_I(I^*)$.

Analogous results arise for an initial excessive capital stock, $K(t_1) > K_0^{+1}$)

3.4 Adjustment to Expected Demand Shifts

3.4.1 Adjustment Paths for Capital and Investment

To examine adjustment to an expected shift the industry level optimization problem is solved at time t_1 when information arises about an expected shift in demand at the later time, t_s .

Equations (3.1) and (3.2) determine the $\dot{I}(t_1)=0$ -curve in Figure 3:10 for $t < t_s$ and the $\dot{I}(t_s)=0$ -curve for $t \geq t_s$. Arrows show the direction of movements before and after the shift.

In order to reach the new stationary state point asymptotically the adjustment path must follow the broken curve in Figure 3:10 from time t_s . Moreover, since the (K, I) -path is continuous for $t > t_1$ (Lemma 3 p. 45), the path must converge to some point on the broken curve as time approaches t_s . Investment must, therefore, shift upwards at time t_1 and follow a northeast movement such that the broken curve is reached smoothly at time t_s . Such a path is illustrated by the solid curve in Figure 3:10. After time t_s , the path follows a southeast move-

-
- 1) The above analysis ignores different types of lags between the time of information and the time of delivery of investment goods. A fixed lag of length z simply means that capital adjustment starts instead at time t_1+z , whereas until then output and output price remain at the short-run equilibrium $(X_s, P_{X,s})$ in Figure 3:8. Analogous straightforward modifications arise for other variables and other cases. For further analyses of investment lags see Thalberg (1960), Jorgenson (1973), Maccini (1973) and Nickell (1978).

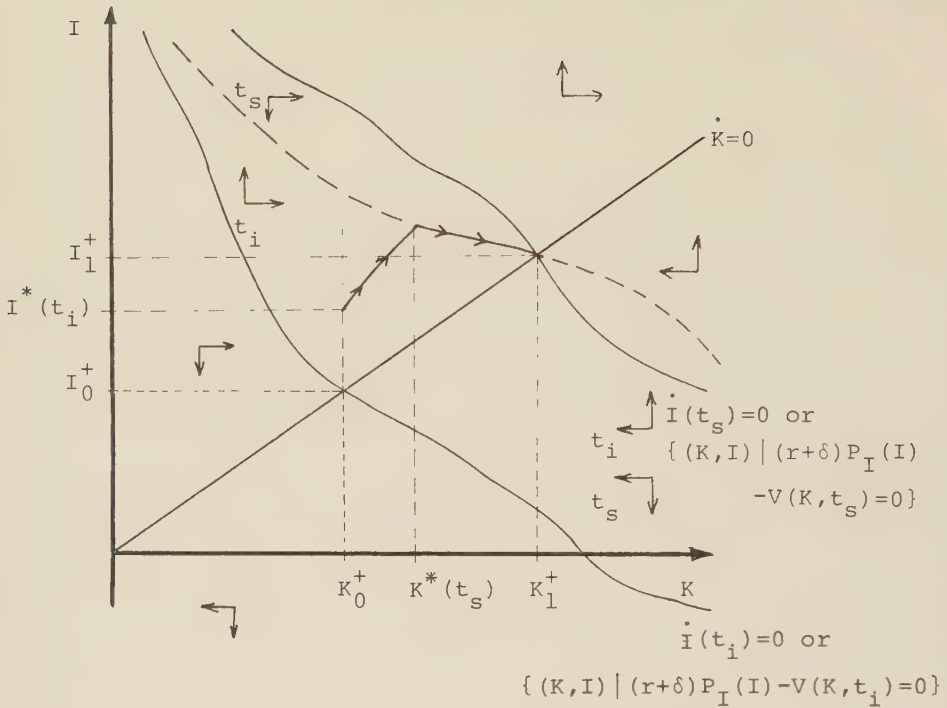


Figure 3:10. Adjustment for an expected demand shift.

ment along the broken curve towards the new stationary state point.

The shift in investment at time t_i is explained by noting that a later departure from the original stationary state results in a capital gain when the interest and depreciation costs of holding capital are equal to or less than the marginal revenue product of capital (since $(r+\delta)P_I(I_0^+)=V(K_0^+, t_i) < V(K_0^+, t_s)$). This violates the well-known necessary condition that the implicit rental for capital services,

$$P_K^*(t) = (r+\delta)P_I(I^*, t) - \dot{I}^*(t)P_I'(I^*, t)$$

equals the marginal revenue product of capital, $V(K^*)$. Less formally, the assumption of a strictly increasing

supply curve for investment goods implies that capital increases through a smooth path of net investment before as well as after the demand shift.

Of course, investment prior to time t_i cannot be increased. An ex post shift in investment may, therefore, occur at t_i in spite of the increasing supply curve for investment goods.

3.4.2 Adjustment Paths for Output and Output Price

The increasing capital before time t_s yields a south-east movement along the original demand curve for the interval $[t_i, t_s]$, as illustrated by arrows in Figure 3:11 (Lemma 2).

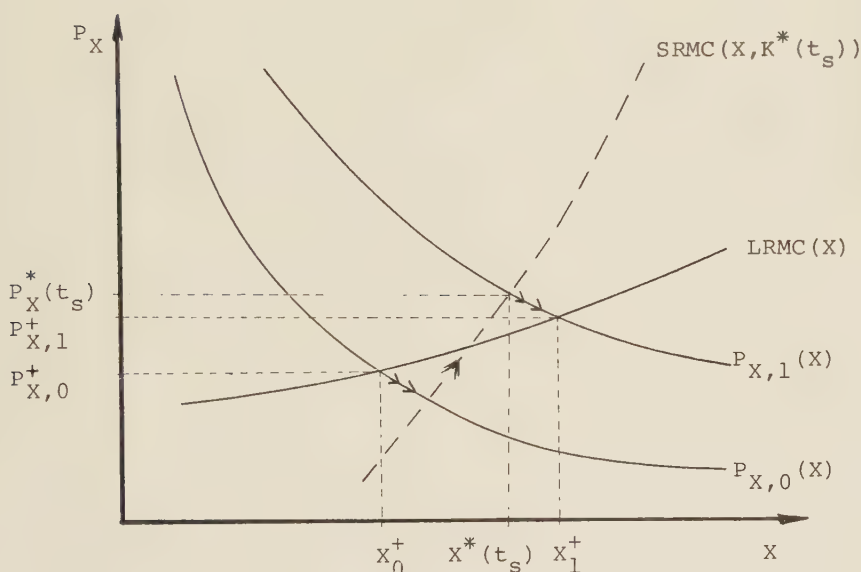


Figure 3:11. Adjustment for an expected demand shift.

At time t_s , the demand shift produces a shift to $(X^*(t_s), P_X^*(t_s))$ along the short-run marginal cost curve for $K^*(t_s)$. The continued capital increase towards the new stationary state position (Figure 3:10) involves a movement along the new demand curve towards the new

long-run equilibrium position, as shown in Figure 3:11.

The adjustment paths for an expected shift is shown by curves $t_s - t_i = 1$ in Figure 3:9.

3.4.3 Conclusions for an Expected Shift

1. The capital adjustment start immediately at t_i involving adjustment in X towards the new stationary state position and in P_X away from its new stationary state value. After shifts in X and P_X at time t_s , K , X , and P_X move monotonically towards their new stationary state levels (Figure 3:9.c-e.).
2. Investment shifts upward at time t_i , increases to a maximum at the time of the demand shift and thereafter decreases towards the new stationary state value (Figure 3:9.b.).
3. This investment path produces a windfall capital gain at t_i , an expected capital gain as a part of the equilibrium return on capital up to time t_s and an expected capital loss thereafter.

Analogous results obtain for an expected downward shift in demand.

3.5 Conclusions for any Demand Shift

In order to examine the effects of a change in the information lead in the next section, we temporarily depart from the assumption of an original stationary state. A comparative dynamics study is undertaken of the general case with any type of upward shift in the marginal revenue product function and any original expected paths for the exogenous factors. The results are also used in Chapter 6. The relevant proposition is as follows:

Proposition 3.3: An upward shift in the marginal revenue product function or a downward shift in the interest rate implies:

- i. strictly higher capital for $t > t_i$
- ii. strictly higher investment for $t \in [t_i, t_s]$
- iii. strictly positive windfall capital gain at time t_i .

A temporary upward shift ending at time t_e in addition implies:

- iv. strictly lower investment for $t > t_e$.

Proof in Appendix p. 165. Illustration in Figure 3:12.

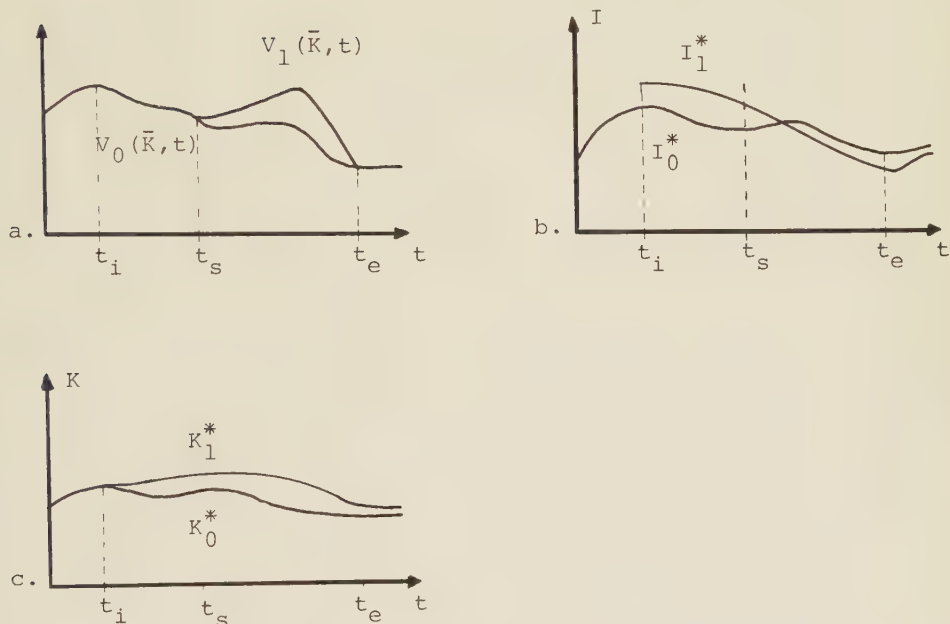


Figure 3:12. Effects of a temporary shift given non-stationary original conditions.

An upward shift in the marginal revenue product function increases the revenue from a marginal investment before the shift, despite the counteracting increases in capital. This results in higher investment before the shift, as illustrated in Figure 3:12.b. The extra

investment ensures that capital subsequently remains higher indefinitely than in the original capital path. This is illustrated in Figure 3:12.c. The higher marginal revenue function tends to induce higher investment during the temporary shift also, but the larger capital stock exerts a counteracting influence. The net effect on investment in the interval $]t_s, t_e[$ is, therefore, uncertain. The effect will change signs in this interval, as illustrated in Figure 3:12.b. After the end of the temporary shift when the investment stimulating effect is eliminated, the outcome is lower investment.

For the case of a basic permanent upward shift in demand and an original stationary states the outcome is that investment increases monotonically up to the shift and decreases monotonically thereafter (Figure 3:9.b). No such conclusions arise for the general case in Proposition 3.3 (Figure 3:12.c).

3.6 A Longer Information Lead

3.6.1 A Longer Information Lead for a Basic Demand Shift

A longer information lead results in higher investment during the additional lead period. Consequently, an expected shift (indicated by 1 in Figure 3:9) implies higher investment than an unexpected shift (indicated by 0 in Figure 3:9) for some time period before the shift (the time period $[t_s-1, t_s[$ in Figure 3:9.b). Higher investment during this period produces a higher capital stock at each subsequent point in time (Figure 3:9.c and Prop. 3.4 below). This higher capital stock induces lower investment during and after the original lead period (Figure 3:9.b and Prop. 3.4). In addition, the larger capital stock leads to higher output and lower output price (Figure 3:9.d-e and Lemma 2).

3.6.2 Perfect Foresight

The limiting case, i.e., when the information lead ap-

proaches infinity, is the case with perfect foresight. This case might be regarded as an optimization at time t_s , with the possibility of backward adjustment. The adjustment paths associated with perfect foresight are shown in Figure 3:9. This limiting case produces no shift in investment with the appealing result that there is no windfall capital gain when foresight is perfect (Prop. 3.4 below.).

3.6.3 A Longer Information Lead for any Demand Shift

The results for the effect of a longer information lead carry over to the general case with any type of shift in the marginal revenue product function or in the interest rate

Proposition 3.4: A longer information lead, i.e.,

$t_{i,2} < t_{i,1}$, on an upward shift in the marginal revenue product function or a downward shift in the interest rate produces:

- i. strictly higher capital for $t > t_{i,2}$
- ii. strictly higher investment for $t \in [t_{i,2}, t_{i,1}]$
- iii. strictly lower investment for $t \geq t_{i,1}$
- iv. strictly lower discounted and undiscounted windfall capital gain.

In the limiting case with perfect foresight, discounted windfall capital gain approaches zero.

Proof in Appendix p. 167.

A longer information lead involves a smoother distribution of extra investment, with higher investment before and lower investment after the end of the period by which the lead has increased. This is illustrated in Figure 3:13. Again, this tendency to smooth investment is explained by the increasing supply price for investment goods.

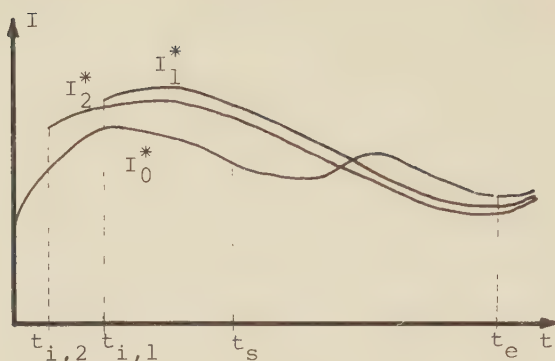


Figure 3:13. Effects of earlier information.

3.6.4 Distributional Effects for Capital Owners and Consumers

The length of the information lead determines the distributional effects of shifts in exogenous factors as between capital owners and consumers. An unexpected upward shift in the marginal revenue product function results in relatively large windfall capital gains for capital owners which are born by consumers through a price in excess of the new long-run equilibrium price for $t > t_s$. This is shown in Figure 3:9.e. A longer information lead implies a lower discounted windfall capital gain for capital owners and a lower output price before as well as after the shift, (Prop. 3.4, Lemma 2 and Figure 3:9.e). In the limiting case with perfect foresight windfall gain is zero and all benefits go to consumers through a low output price path.

A longer information lead for a downward shift in the marginal revenue product function results analogously in a lower discounted windfall capital loss for capital owners. Consumers lose through a higher output price. In the limiting case windfall capital loss is zero and all costs are born by consumers through a high output price path.

3.7 Determinants of the Adjustment Speed

3.7.1 Introduction

This section analyzes the determinants of the speed of adjustment between two long-run equilibrium positions. We analyze the conditions under which the market adjusts slowly, as illustrated by curves 1 in Figures 3:14 and 3:15. This is compared with a case involving relatively rapid adjustment as illustrated by curves 2.

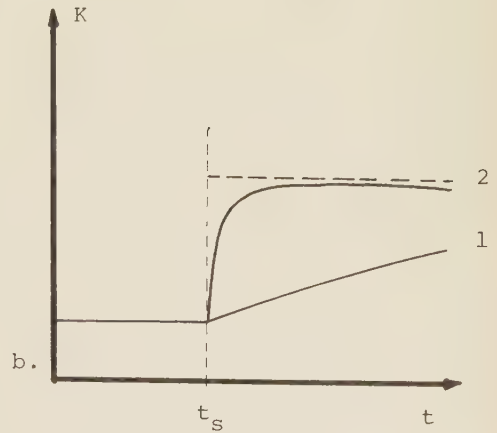
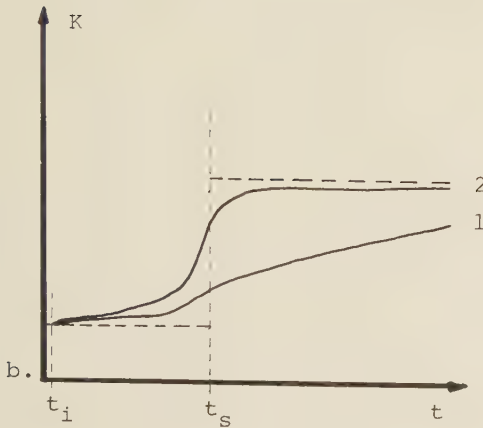
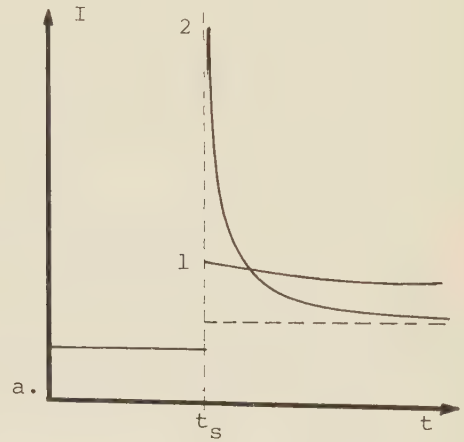
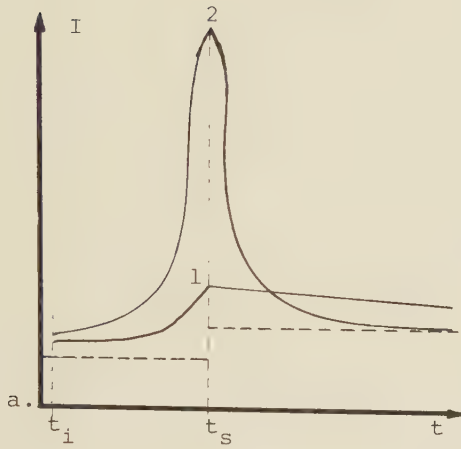


Fig. 3:14. Adjustment paths for an expected shift.

Fig. 3:15. Adjustment paths for an unexpected shift.

The speed of adjustment is interesting for at least three reasons:

1. In order to predict and explain the effects of a change in an exogenous factor we need to know how quickly the market adjusts to the new long-run equilibrium position. If adjustment is very rapid the comparative statics model with instantaneous adjustment will provide a reasonable approximation of the effect of a shift. Conversely, if adjustment is slow the static model gives a very poor picture of the adjustment process and this points to the need for an explicit dynamic analysis.
2. Slower adjustment increases the relative importance of the distant future, thereby producing more forward-looking investment. This is further analyzed in Chapter 6 below.
3. Faster adjustment tends to induce stronger investment fluctuations, as illustrated in Figures 3:14-3:15. This is further analyzed in Chapter 8.

3.7.2 The Relationship between Deviations in Investment and Capital Stock

In order to examine the determinants of the adjustment speed we note that (2.15) implies:

$$\begin{aligned}
 (3.7) \quad & P_I(I^*(t), 0) - P_I(I^+(t), 0) = \\
 & = \int_t^{\infty} (V(K^*(s), s) - V(K^+(s), s)) e^{-(r+\delta)(s-t)} ds
 \end{aligned}$$

Stationary state investment and capital as well as the V-function shift at time t_s . This explains the time indexes in (3.7).

Equation (3.7) states that the current departure of investment price from its stationary state value is equal to the discounted and depreciated sum of future departures of the marginal revenue product from its stationary state values. Less formally, excessive investment price is equal to the sum of the future excessive marginal revenue products.

Since $K^*(s) > K^+(s)$ for $t_i < s < t_s$, it follows that $V(K^*(s), s) - V(K^+(s), s) < 0$ for $t_i < s < t_s$.

Opposite inequalities hold for $t > t_s$ (Figure 3:9.e). Since $I^*(t) > I^+(t)$, the positive terms on the right hand side of (3.7) after time t_s dominate the negative terms before t_s .

3.7.3 Effects of Slopes of Crucial Functions on Adjustment Speed

For a given value of the right hand side in (3.7) a higher slope of the supply function for investment goods, P_I' , implies a smaller deviation of I^* from I^+ . This is illustrated in Figure 3:16.

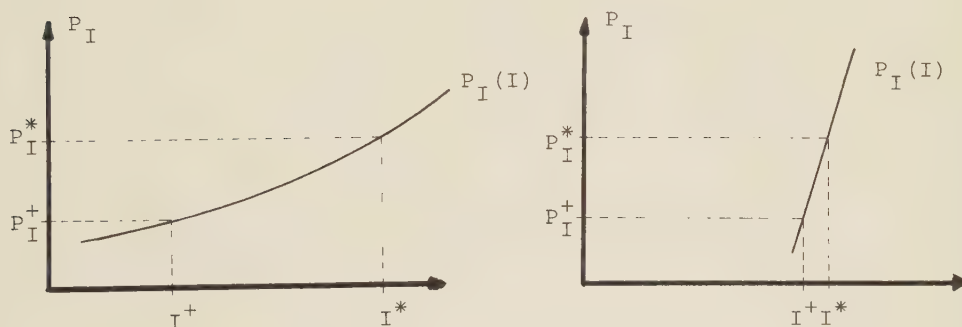


Figure 3:16. Investment deviations for different slopes of the supply curve for investment goods.

If P_I' is very high relative to the absolute slope of the $V(K)$ -function, $|V'|$, (3.7) requires I^* to be fairly close to I^+ at all points in time. K^* is, therefore, fairly close to K^+ before time t_s and relatively far from K^+ thereafter. A high value of P_I' therefore produces relatively slow adjustment and relatively smooth capital and investment paths. This is illustrated by paths 1 in Figures 3:14-3:15. Equation (3.7) also reveals that a lower P_I' or a higher $|V'|$ gives larger deviations of I^* from I^+ and smaller deviations of K^* from K^+ and, consequently, faster capital adjustment.

This is illustrated by the shift from paths 1 to paths 2 in Figures 3:14-3:15.

In the limiting case where P_I' approaches zero the supply curve for investment goods in Figure 3:16 is a horizontal line. This implies that the price of investment goods is exogenous for the industry. As concluded in Section 2.7, capital then adjusts instantaneously through infinite investment at time t_s . Investment equals its respective stationary state level before and after time t_s . The rapid adjustment shown by path 2 becomes successively more pronounced as P_I' approaches zero.

The speed of adjustment is decreasing in P_I' , with an infinite speed for $P_I'=0$. As an intuitive explanation, a steep supply curve results in smooth investment, since high investment then involves a very high price and low investment involves a very low price, thereby precluding extreme levels of investment. An industry purchasing its investment goods on a domestic market with a relatively steep supply curve is consequently predicted to adjust slower than an industry purchasing its investment goods on an international market with a flat supply curve.

The speed of adjustment is increasing in $|V'|$. Again intuitively, a steep V-function implies a high marginal revenue product for $K^* < K^+$, thereby inducing high investment around time t_s . There is, therefore, fast and concentrated adjustment around this point of time. A higher elasticity of substitution between K and L and a higher absolute price elasticity of demand result in a smaller decrease in V for a given increase in K, i.e. $|V'|$ decreases. These changes lead, therefore, to slower adjustment. For example, an industry in a small country selling its output on an international market with a high absolute price elasticity of demand is predicted to adjust more slowly than an industry selling its output on a domestic market with a low absolute price elasticity of demand. The explanation is that K

in excess of K^+ has a less serious effect on the marginal revenue product of capital when output is sold on an international market since extra output is absorbed with a relatively small price decrease. This dampens the adjustment speed. Analogously, a higher elasticity of substitution means that an excessive K gives rise to a smaller decrease in marginal revenue product, thereby causing slower adjustment.

In the limiting case when $|V'|$ approaches zero, the right hand side in (3.7) tends to zero. Investment then shifts from the original to the new stationary state value at time t_g and adjustment is very slow. The over-adjustment in investment implied by our assumptions (Figure 3:9.b) is thus eliminated in this limiting case.

In addition to the above effects, a higher P_I' produces a smaller change in stationary state investment for a given demand shift. This strengthens the tendencies to smooth investment. In the limiting case with a vertical supply curve stationary state investment is unaffected by the demand shift. Investment and capital stock are then constant over time. The classical example is land fixed in supply, for which a demand shift affects only price.

A higher $|V'|$ induces a smaller change in stationary state investment and capital stock. This dampens the tendency to larger variations in investment associated with more rapid adjustment caused by higher $|V'|$. This is analyzed further in Chapter 8.

3.7.4 Effects of Interest and Depreciation on Adjustment Speed

A higher $(r+\delta)$ raises the total discount factor on the right hand side of (3.7). Deviations in investment are then more closely related to marginal revenue product deviations in the near future. A higher $(r+\delta)$ leads, therefore, to faster adjustment and larger changes in investment, corresponding to the shift from path 1 to 2 in Figures 3:14-3:15.

A higher δ increases the speed of adjustment not only through its influence on the total discount factor, but also through another effect. To see this note that the definition of net investment implies:

$$(3.8) \quad \dot{K}/K = \delta \left(\frac{(I/K) - (I^+/K^+)}{(I^+/K^+)} \right)$$

The equation shows that a higher depreciation rate induces a higher rate of change in capital stock for a given percentage deviation of the I/K -quotient from its stationary state value. As an explanation, a higher depreciation rate involves higher replacement investment. A given percentage change in gross investment then has a stronger effect on net investment. This effect does not obtain for the interest rate.

In the limiting case when δ approach infinity, (3.7) reduces to $P_I(I^*, 0) = V(I^*, t)$ and $K^* = I^*$. Capital then becomes a nondurable input, where input price is equal to marginal revenue product at each point in time. Capital and investment adjust instantaneously at time t_s . The special characteristics of a durable input are eliminated and, consequently, the problem of adjustment disappears.

3.7.5 Effects of Long-run Growth Rate on Adjustment Speed

As analyzed in 3.2.3, the depreciation rate in our model is the sum of the depreciation rate for basic capital and the long-run growth rate; $\delta = \delta_b + g_1$. A higher

long-run growth rate, therefore, increases the adjustment speed. As an intuitive explanation, a higher long-run growth rate of demand implies a higher normal I/Y -quotient. Again, this implies that a certain percentage change in this quotient has a stronger effect on net investment (Equation 3.8).

3.7.6 An Interpretation

The degree of smoothness in the capital path is determined by two counteracting factors. On the one hand, capital tends to be close to its current and near-future stationary state level. Any lower capital stock, for example, has a marginal revenue product which is high relative to the stationary state marginal cost of investment, $(r+\delta)P_I(\delta K^+, t)$. This induces high investment.

On the other hand, the increasing price of investment goods tends to smooth the investment path. This brings about a smooth capital path with large capital deviations from its current and near-future stationary state levels.

The determination of the optimal capital path is essentially a compromise between the desirability of having K^* close to current and near-future K^+ and the desirability of having smooth investment. Interpreted in these terms, the above results may be summarized as follows:

1. A higher slope for the V -function strengthens the tendency for capital to be close to its current and near-future stationary state level and, therefore, results in faster adjustment.
2. A higher slope for the P_I -function reinforces the tendency to smooth investment path, with associated slower adjustment.
3. A higher $(r+\delta)$ increases the relative importance of stationary state capital in the very close future, and, therefore, increases the speed of adjustment.
4. A high δ in addition implies that large net investment requires a relatively small percentage increase in gross investment. This further strengthens the tendency to

rapid adjustment.

3.7.7 Numerical Results for the Adjustment Speed

We now derive numerical results for the adjustment speed in order to examine, first, the speed of adjustment for reasonable parameter values and, secondly, the sensitivity to changes in the different determinants.

As is shown in Section 6.5 below, linear approximations of the $P_I(I)$ - and $V(K)$ -functions around their new stationary state values yield the following expression for the share of the remaining capital adjustment for $t \geq t_s$:

$$(3.8) \quad \frac{K^*(t) - K^+}{K^*(t_s) - K^+} = e^{-\left[\sqrt{(r/2)^2 + (r+\delta)\delta(1+|V^+ - e|)/P_I^+ - e|} - r/2 \right] (t - t_s)}$$

$$\text{where } V^+ - e| \triangleq \frac{\partial \ln V(K^+)}{\partial \ln K} \quad \text{and} \quad P_I^+ - e| \triangleq \frac{\partial \ln P_I(I^+)}{\partial \ln I}$$

Equation (3.8) shows that the distance to the new stationary state capital in Figures 3:14-3:15 decreases exponentially after time t_s .

Consequently, the time required after time t_s to close half of the gap between actual and stationary state capital at time t_s is given by:

$$(3.9) \quad t - t_s = [-\ln 0.5] / \left[\sqrt{(r/2)^2 + (r+\delta)\delta(1+|V^+ - e|)/P_I^+ - e|} - r/2 \right]$$

Figure 3:17 shows the length of this period for reasonable parameter values.

The figure supports the earlier conclusions that the speed of adjustment increases in δ , r , and $|V'|$ and decreases in P_I' .

Figure 3:17 reveals surprisingly slow adjustment. For capital with a 10% depreciation rate, half of the adjustment takes about 4 years, given the above parameter ranges.

The speed of adjustment is revealed to be highly sensitive to changes in the depreciation rate for unchanged

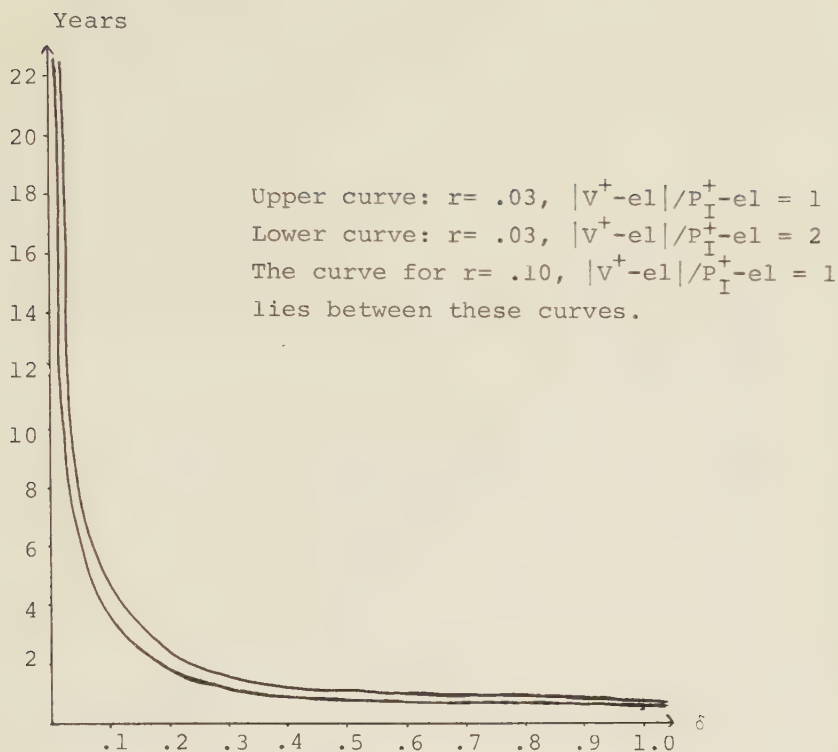


Figure 3:17. Time in years to close half of the gap between actual and stationary state capital at time t_s .

r and $|v^+_{el}|/P^+_{I-el}$. Given the assumed ranges for interest and the relative elasticities it is predicted that half of the adjustment will occur in 1 year for machinery and equipment with $\delta = .5$, in 2-4 years for ships and cars with $\delta = .1-.2$ and in about 10 years for houses with $\delta = .03-.05$.

The interest rate and the relevant elasticities yield relatively small modifications for the considered parameter ranges.

A change in the real interest rate from .03 to .10 is predicted to decrease the period for half of the adjustment from 4.7 to 4.4 years for $\delta = .1$ and $|v^+_{el}|/P^+_{I-el} = 1$. An industry such as car rentals with a high rate of

discount, as a result of high taxes on capital income, is predicted, therefore, to adjust somewhat faster than the market producing services from privately owned cars, where the rate of return is low as a result of no taxes on imputed income from personal durables.

Whereas an increase in r from .03 to .10 decreases the studied period from 4.7 to 4.4, the same absolute increase in δ from .1 to .17 decreases the period from 4.7 to 2.2. This illustrates the point that the adjustment speed is more sensitive to changes in δ than to changes in r .

A higher $|V^+ - el|$ causes faster adjustment. The quantitative analysis indicates moderate effects. For the basic case with $|V^+ - el|/P^+ - el = 1$, $r = .03$, and $\delta = .1$, a double elasticity decreases the period from 4.7 to 3.8 years, while an elasticity ten times greater decreases the period to 1.9 years.

Since only the quotient between the absolute V^+ -elasticity and the P_I^+ -elasticity matters in (3.9), a change to half of the basic P_I -elasticity also decreases the period for half of the adjustment from 4.7 to 3.8 years for the basic parameter values. A change to one tenth of the basic P_I -elasticity decreases the time to 1.9 years.

3.8 Rational Versus Stationary Expectations

This section compares adjustments under rational and stationary price expectations. Since stationary price expectations are applicable to an unexpected shift only, the comparison is restricted to this special case.

Under stationary price expectations each firm expects P_X and P_L to remain constant over time. The marginal productivity of labor is determined by the condition $F'_L = P'_L(t)/P_X(t)$. For our homogeneous of degree one production function the marginal productivities of K and

L are determined by the quotient K/L only. Stationary price expectations, therefore, produce the result that F'_L , K/L and F'_K are expected to remain constant over time. Consequently, the marginal revenue product of capital, $V(t)=P_X(t)F'_K$, is also expected to remain constant. Investment at time t is then determined by the following condition (from (2.15)):

$$(3.10) \quad P_I(I, t) = V(K, t) / (r(t) + \delta).$$

Under the assumption of constant exogenous factors after time t_s , this simplifies to:

$$(3.11) \quad P_I(I, t_s) = V(K(t), t_s) / (r(t_s) + \delta)$$

The $\dot{I}(t_s)=0$ -curve in Figure 3:18 is determined by the same condition. Investment at time t is, therefore, determined by the point on this curve for $K(t)$. An un-

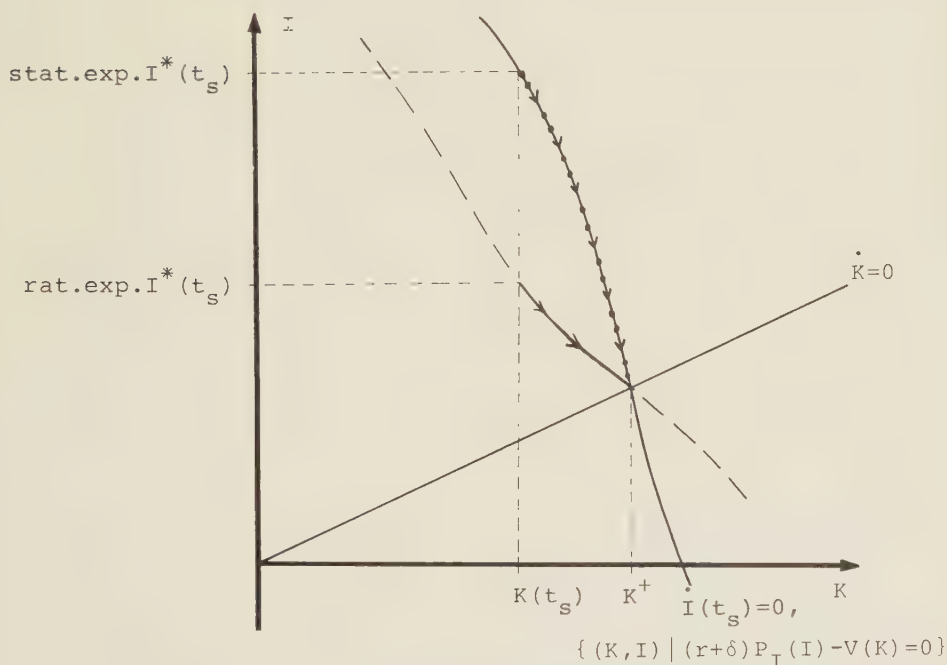


Figure 3:18. Adjustments to an unexpected demand shift under stationary and rational expectations.

expected shift in demand induces a shift in investment to $\text{stat.exp.}I(t_s)$ in Figure 3:18. Net investment is positive northwest of the $\dot{K}=0$ -line producing an increa-

sing capital. Stationary expectations bring about southeastward adjustment along the $\dot{I}=0$ -curve as shown by the dotted curve in Figure 3:18.

Under the assumption of rational price expectations, investment is determined by the condition (from (2.15')):

$$(3.12) \quad P_I(I^*, t_s) = (V(K) + \dot{I}^*(t) P_I'(I^*, t_s)) / (r(t_s) + \delta),$$

The resulting adjustment is shown by the solid curve in Figure 3:18.

A comparison of (3.11) and (3.12) shows that investors take account of the systematic capital losses during the adjustment process under rational but not under stationary price expectations. This explains the lower investment and, consequently, the slower adjustment under rational expectations.

3.9 Static and Dynamic Optimization

Under the assumption of rational price expectations current investment is determined as a part of a dynamic optimization problem. The solution to this problem determines an entire future capital path, $\{K^*(t) \text{ for } t \geq t_p\}$. It is apparent that current investment depends on the entire time path for the marginal revenue product of capital and, consequently, on the future capital path (from 2.15):

$$(3.13) \quad P_I(I^*, t_p) = \int_{t_p}^{\infty} V(K^*(s), s) e^{-\delta(s-t_p) - R(s, t_p)} ds$$

There is, therefore, a mutual interdependence between investment and the marginal revenue product of capital. On the one hand, investment depends on future marginal revenue product of capital and, on the other, investment determines future capital stock and, consequently, the future marginal revenue product of capital. Current investment thus depends on future investment, i.e. current investment is determined as a part of an investment plan for the entire future, $\{I^*(t) \text{ for } t \geq t_p\}$.

Earlier models for adjustment and investment determination, such as those of Keynes (1936), Lerner (1944), Clower (1954b), Eisner-Strotz (1963), Wijkman (1965), Smith (1966), Lucas (1967a,b), Gould (1968) and Treadway (1970, 1971, 1974) determine investment through static optimization in the sense that current investment is independent of future investment. This simplification arises through the highly restrictive assumption that future marginal revenue product of capital is independent of future capital! The equation determining investment then reduces to:

$$(3.14) \quad P_I(I, t_p) = \int_{t_p}^{\infty} V(s) e^{-\delta(s-t_p) - R(s, t_p)} ds,$$

where future marginal revenue product, $V(s)$, is independent of future capital.

Under stationary expectations $V(s)$ is equal to $V(K(t_p), t_p)$. Investment is then determined through the static optimization analyzed in the preceding section.

If the price paths for output and the nondurable input are exogenous¹⁾ the marginal productivity of the nondurable input is determined by the condition $F'_L = P_L(s)/P_X(s)$. For a homogeneous of degree one production function this condition determines K/L , F'_K and $V(s) = F'_K P_X(s)$ from the exogenous prices $P_L(s)$ and $P_X(s)$. The marginal revenue product is, therefore, independent of the future capital path and, consequently, investment is determined through the static optimization of equation (3.14).²⁾

Other models³⁾ assume that future marginal revenue

1) Eisner-Strotz (1963), Lucas (1967a), Gould (1968) and Treadway (1970, 1971).

2) Wijkman (1965) does not assume that the production function is homogeneous on degree one. His implicit assumption of stationary expectations for P_X, P_L , and P_I means that investment is determined as in Lucas' static model.

3) Lerner (1944), Clower (1954b), Smith (1966).

products depend on current capital and possibly time, i.e., $V(s) = V(K(t_p), s)$, while no explicit analysis is made for the determination of the future marginal revenue product. Alternatively, marginal revenue product is assumed to be determined by current investment and time, i.e., $V(s) = V(I(t_p), s)$,¹⁾ with investment again being determined in the static optimization of Equation (3.14).

The above static models are inadequate for an analysis of market adjustment where prices depend on capital stock and investors recognize this dependence. The step from static to dynamic optimization allows for a theory in which investors recognize this dependence.

A main innovation in the current investment theory is that our dynamic optimization model determines the entire future capital path and thereby allows for a mutual interdependence between prices and capital stock. Earlier investment theories based on static optimization models typically excluded most of the influence of future capital on future prices.²⁾

This extension in the current study is analogous to that made by Arrow-Kurz (1970). In their model the path for aggregate savings for an economy is determined in a dynamic optimization model in contrast to earlier static optimization models in which systematic mistakes are made during the adjustment process.³⁾

1) Keynes (1936), cf. Section 6.6.

2) The mutual interdependence is also recognized by Nickell (1974, 1975, 1978) for the case with monopoly analyzed in Chapter 7 below, by Arvidsson (1960) and Treadway (1969) for the case with exogenous price paths and in the intertemporal general equilibrium model, Malinvaud (1953).

3) Fisher (1930), Lerner (1944), Friedman (1962) and Uzawa (1969).

3.10 Successive Information

In the above analysis information on a future demand shift arose at a single point of time, t_1 , leading to an instantaneous shift in expected demand after t_s . This once and for all shift in expectations is illustrated by the solid line in Figure 3:19a. It is possible, however, that firms instead obtain successively more reliable information on the future demand shift, producing successive adjustments in expected demand for time after t_s . This adjustment of expectations is illustrated by the dotted curve in Figure 3:19.a.

Each time new information is received the firms optimization problems are solved for the new expected paths for the exogenous factors, with consequent successive revisions of investment plans. The expected $I=0$ -curve after time t_s in Figure 3:19.b shifts successively, producing an ex post adjustment path of the type shown by the dotted curve. In contrast to the case with instantaneous information, the lower expected demand for some time after t_1 yields lower investment at t_1 and lower capital at each later point in time. This is illustrated in Figures 3:19.c-d (Prop. 3.3.).

As before, new information causes a shift in investment and windfall capital gains. Such gains need not, however, be inconsistent with rational expectations. The windfall gains might ex ante be balanced by the windfall losses that would have arisen if the new information had instead led to a downward adjustment in future expected demand.

The main effect of successive information is to delay the capital adjustment, as illustrated in Figure 3:19.c. The speed of adjustment of expected demand, therefore, enters the analysis as an additional determinant of the speed of capital adjustment. More rapid adjustment of expected demand causes faster capital adjustment. The lower investment before time t_s gives rise to less smooth investment through higher investment after time t_s .

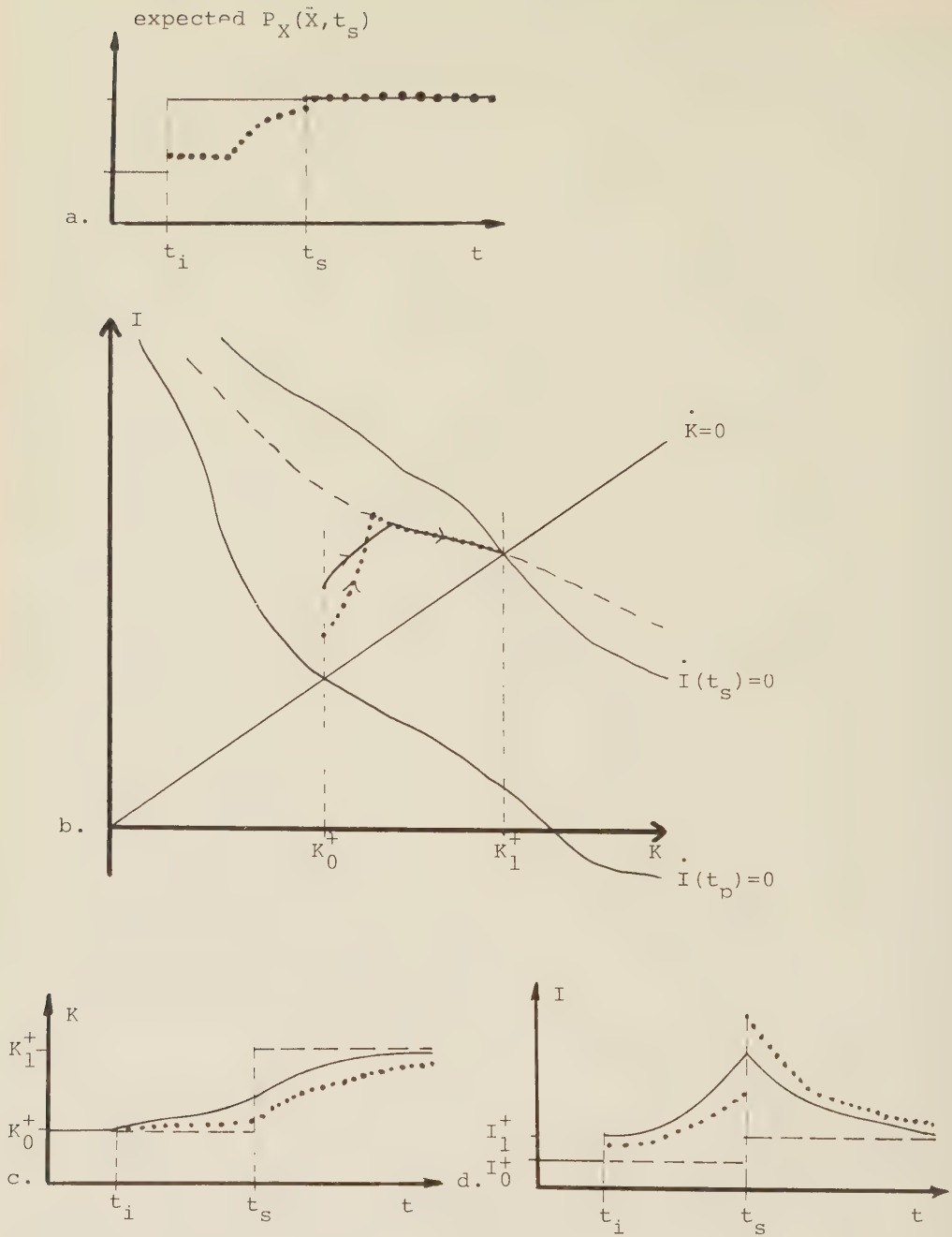


Figure 3:19. Adjustments to an expected shift with successive (dotted curves) and instantaneous (solid curves) information.

4. ADJUSTMENTS TO PERMANENT SHIFTS IN OTHER FACTORS

4.1 Introduction

The industry level optimization problem (2.6) determines the industry's ex ante investment path for any expected time paths for the exogenous factors:

$\{P_X(X,t), P_I(I,t), P_L(L,t), F'_K(K,L,t), F'_L(K,L,t), r(t) \text{ for } t \geq t_p\}$.

To analyze shifts in technology the industry level production function is split into two parts, using the Euler-equation:

$$(4.1) \quad F(K,L,t) = K F'_K(K,L,t) + L F'_L(K,L,t).$$

Chapter 3 analyzes adjustment to a basic permanent shift in $P_X(X,t)$ from an original stationary state. This chapter analyzes the effect of basic permanent shifts in the other exogenous factors.

4.2 Shifts in Labor Supply, Technology and Interest

4.2.1 Adjustment Paths for Capital and Investment

An expected basic permanent shift in the supply curve for labor is illustrated in Figure 4:1.

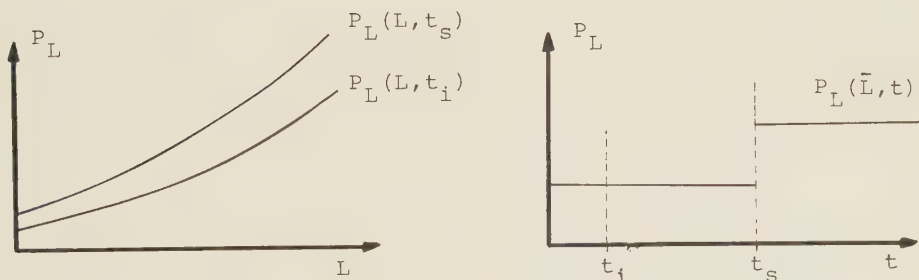


Figure 4:1. Basic upward shift in $P_L(L)$

An upward shift in $P_L(L)$ reduces L for a given $\bar{K}$. This results in a higher marginal revenue product for a gi-

ven capital through the first factor in the following expression:

$$(4.2) \quad V(\bar{K}, t) = P_X(F(\bar{K}, G(\bar{K}, t))) F'_K(\bar{K}, G(\bar{K}, t))$$

Since $F''_{KL} > 0$, the second factor has a counteracting effect. For a sufficiently high elasticity of substitution the decrease in F'_K is small enough to be dominated by the increase in P_X . An upward shift in $P_L(L)$ then produces an upward shift in the $V(K)$ -function. A high absolute price elasticity of demand gives rise to a small price increase and, hence, a weaker tendency for a higher $V(\bar{K})$. More specifically, the increase in P_X dominates the decrease in F'_K , thereby shifting the $V(K)$ -function upwards, if and only if the elasticity of substitution exceeds the absolute price elasticity of demand, i.e., if $(\sigma + \epsilon) > 0$. (Proof in Lemma A.3 in Appendix). Restricting the analysis to this case, an upward shift in $P_L(L)$ leads to a northeast shift in the $I=0$ -curve as illustrated in Figure 4:2. This results in upward shifts in stationary state capital and investment. (See also Prop. 4.1 below).

The adjustment is illustrated by the solid curve starting at K_0^+ and converging to the new stationary state. The determination of the adjustment paths for capital and investment in response to a shift in $P_L(L)$ is, indeed, equivalent to the case with a demand shift. This is apparent since capital and investment depend only on the time paths $\{V(K, t), P_I(I, t), r(t) \text{ for } t \geq t_p\}$ in Equations (2.15)-(2.17). Shifts in different exogenous factors which produce the same type of shift in the $V(K)$ -function consequently induce the same type of adjustment.

An upward shift in $P'_K(K, L)$ results in a higher marginal revenue product through the second factor in (4.2).

The tendency to a higher output exerts a counteracting effect through the first factor. If

$[P_X F''_{LL} (1 + \epsilon) + \frac{P_X F'_L F'_L}{X} - P'_L (\frac{KF'_K}{X} + \epsilon)] > 0$, which is, for example, the case for $\epsilon < -1$, the higher F'_K dominates.

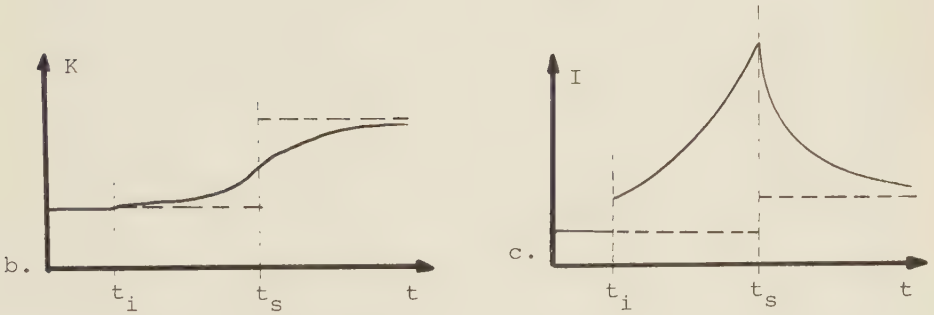
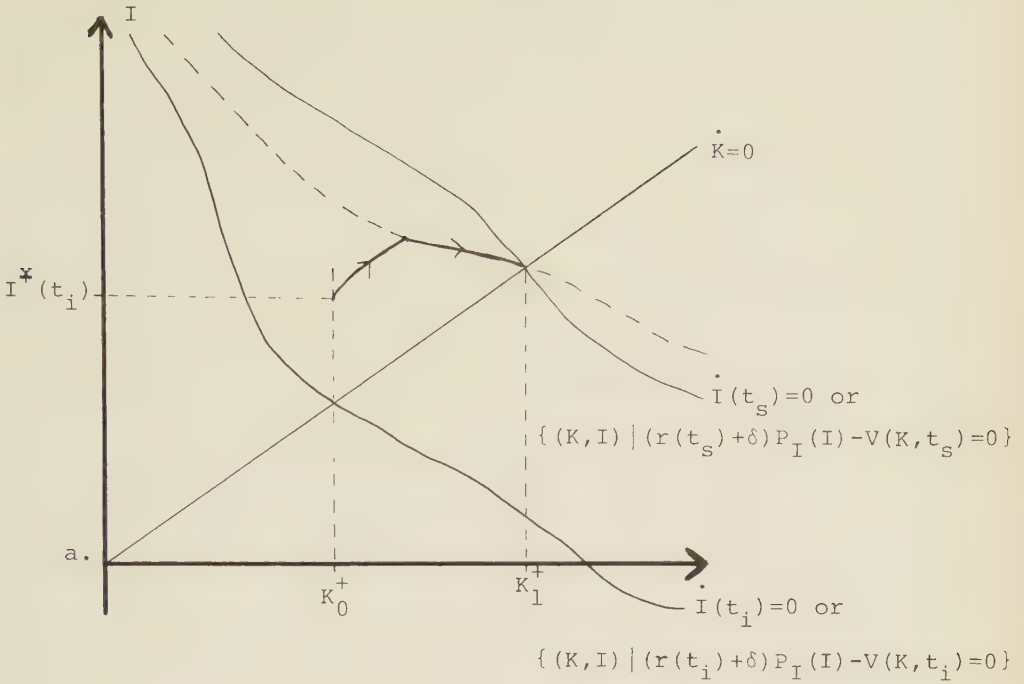


Figure 4:2. Adjustments to upward shifts in $P_L(L)$ or $F_L^K(K, L)$ and downward shifts in $F_L^K(K, L)$ and r .

This results in an upward shift in the $V(K)$ -function and, hence, in higher stationary state capital (Figure 4:2, Lemma A.3 and Prop. 4.1). The adjustment paths for capital and investment are again given by the solid curves in Figure 4:2.

A downward shift in $F'_L(K,L)$ produces an upward shift in $V(K)$ for a sufficiently strong substitution effect or a sufficiently small absolute price elasticity of demand as shown by the condition

$[1 + \sigma + \epsilon + (F'_K P'_L / F''_{KL} P_X X)] > 0$. If this condition is satisfied, which is the case for $(\sigma + \epsilon) > -1$, the adjustment to a downward shift in $F'_L(K,L)$ is shown in Figure 4:2. (Lemma A.3 and Prop. 4.1.)

Under the opposite inequality sign for the respective conditions, the shifts in $P_L(L)$, $F'_K(K,L)$ and $F'_L(K,L)$ lead instead to a lower stationary state capital with an analogous determination of the adjustment path. (Prop. 4.1.)

Finally, the important case with a downward shift in r also gives rise to a rightward shift in the $I=0$ -curve in Figure 4:2. This induces a higher stationary state capital and the adjustment shown in Figure 4:2. (Prop. 4.1.)

Shifts in $P_X(X)$, $P_L(L)$, $F'_K(K,L)$, $F'_L(K,L)$ and r lead, therefore, to the same type of adjustment in K and I . It follows that the earlier results for the effect on capital and investment of unexpected and expected shifts (Prop. 3:3), earlier information (Prop. 3:4), and the determinants of the adjustment speed (Section 3.7) hold for these shifts also.

4.2.2 Adjustments in Output, Output Price and Labor

It is now demonstrated that shifts in $P_L(L)$, $F'_K(K,L)$, $F'_L(K,L)$ and r result in different adjustment paths for X , P_X and L .

An upward shift in $P_L(L)$ or a downward shift in $F'_L(K,L)$ induces an upward shift in the long-run marginal cost curve. This is illustrated in Figure 4:3. (Formal proof in Prop. 4.1 below.)

The growth in capital before time t_s leads to a south-east movement along the demand curve from time t_i to time t_s , as illustrated in Figure 4:3.a-c. (Lemma 2.)

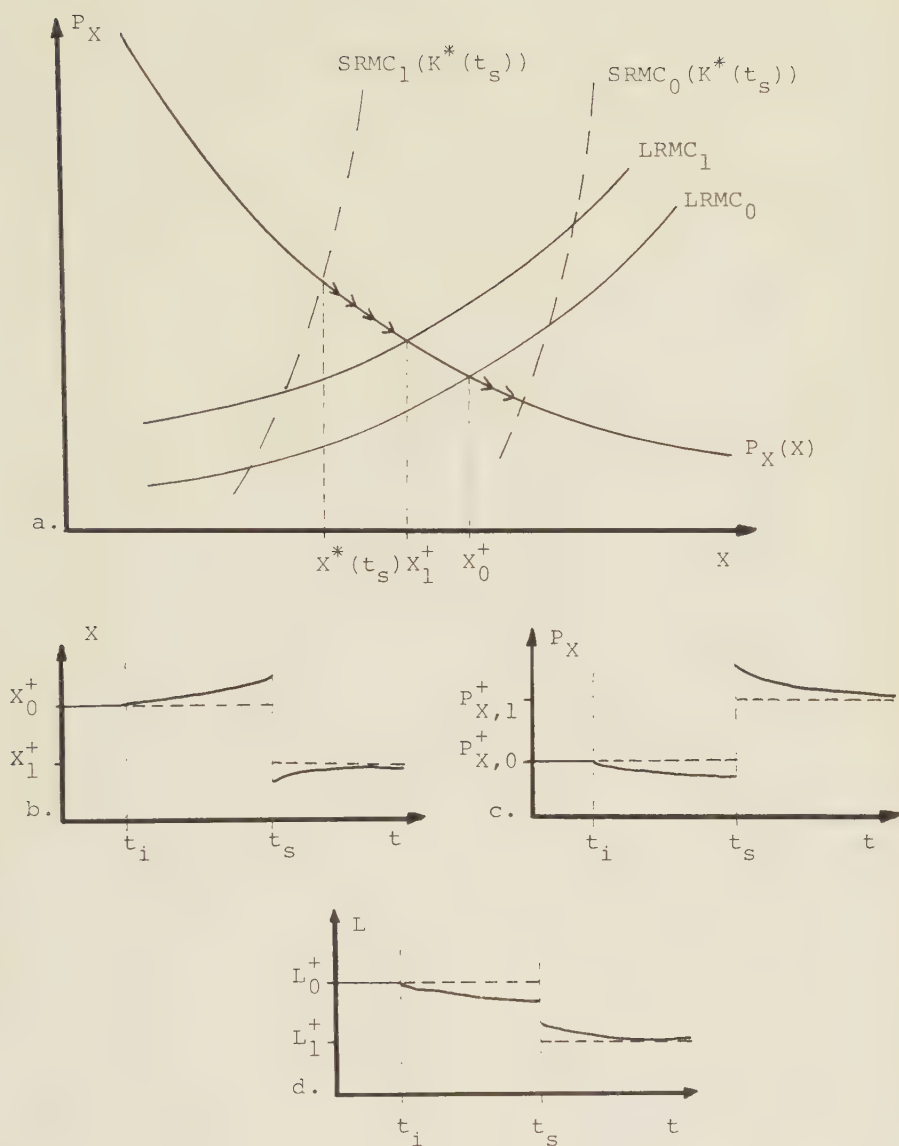


Figure 4:3. Adjustments to an upward shift in $P_L(L)$ or a downward shift in $F'_L(K,L)$.

At time t_s , the upward shift in $P_L(L)$ or the downward shift in $F'_L(K,L)$ produces an upward shift in the short-run marginal cost curve for $K^*(t_s)$. This results in a downward overshooting shift in output and an upward overshooting shift in output price, as shown in Figure 4:3.a-c.

The continued increases in capital in Figure 4:2 cause a continued increase in output and decrease in output price after time t_s (Lemma 2). This is shown in Figure 4:3.a-c.

Where the substitution effect dominates (i.e. $(\sigma+\epsilon)>0$), capital increases before and after time t_s cause decreases in L both before and after time t_s (Lemma 2). This results in the path shown in Figure 4:3.d for the case with a downward shift in stationary state labor. If the substitution effect dominates for the $F'_L(K,L)$ -shift but not for the K -shift (i.e. if $(1+\sigma+\epsilon+(F'_K P'_L L/F''_{KL} P_X X))>0$ and $(\sigma+\epsilon)<0$) the increasing capital instead gives rise to an increasing L both before and after time t_s , with a consequent overshooting at time t_s in Figure 4:3.d.

An upward shift in $F'_K(K,L)$ is reflected in a downward shift in the long-run marginal cost curve, as illustrated in Figure 4:4.a (Prop. 4.1). The increasing capital in Figure 4:2 implies a southeast movement along the demand curve both before and after time t_s . The shift in $F'_K(K,L)$ induces a southeast shift at t_s , as shown in Figure 4:4.a. The paths for X , P_X and L for a dominating substitution effect are shown in Figure 4:4.b-c.

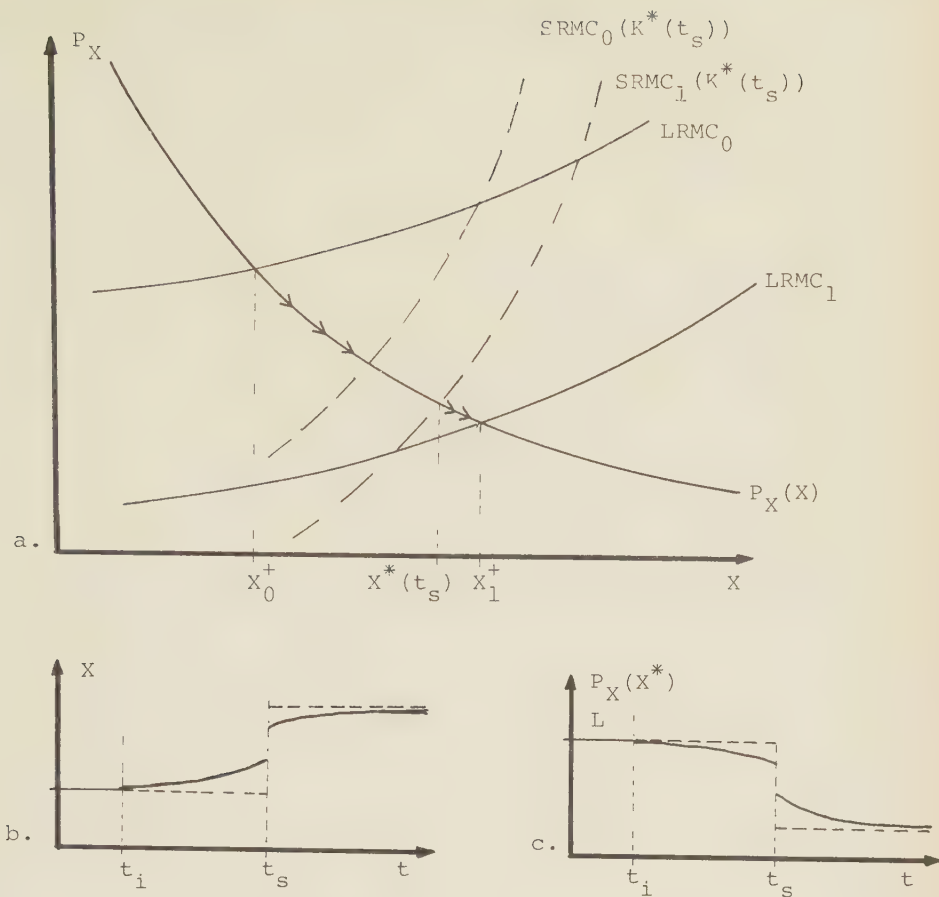


Figure 4:4. Adjustment to an upward shift in $F'_K(K, L)$.

Finally, a downward shift in r produces a downward shift in the long-run marginal cost (Prop. 4.1). In contrast to earlier cases a shift in r does not shift the short-run marginal cost curve. Instead X , P_X , and L adjust smoothly and monotonically from the original to the new long-run equilibrium position, as shown in Figure 4:5.

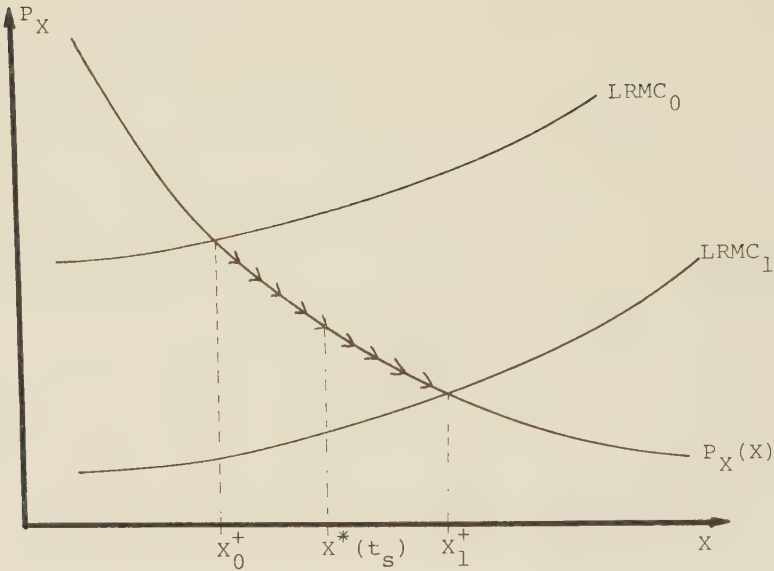


Figure 4:5. Adjustment to a downward shift in r .

4.2.3 An Application

In order to illustrate the significance of the above results, the analysis is used to examine the effect of a tax change that, say, decreases taxation on business capital. Such a tax reform has been implemented in several European countries.

For a given after-tax rate of return, r_a , a decrease in the tax rate from tax_0 to tax_1 decreases the before-tax rate of interest from $r_a/(1-\text{tax}_0)$ to $r_a/(1-\text{tax}_1)$.¹⁾

The above model predicts that the tax reform will give rise to an upward shift in investment and a windfall capital gain at the time of information (Prop. 3.3).

1) In a general equilibrium model r_a typically changes too. For the current analysis it is sufficient to assume that the net effect is dominated by the tax change.

The expected tax decrease induces higher and increasing investment with expected capital gains up to the time of the tax decrease (Figure 4:2).¹⁾ After the tax change investment decreases monotonically towards the new stationary state level with expected capital losses occurring during this part of the adjustment (Figure 4:2).

Higher investment means a larger capital stock after the time at which the information emerges (Figure 4:2). This in turn leads to higher X , lower P_X and lower L , where the substitution effect dominates. The smooth capital adjustment implies smooth and monotonic X -, P_X - and L -adjustments from the original to the new long-run equilibrium (Figure 4:5).

The analysis also suggests that the length of the information lead is a crucial determinant of the size of the windfall capital gain and, therefore, of the distributional effects as between capital owners and consumers. The higher capital stock associated with a longer information lead decreases the (discounted) windfall capital gain by passing on the gain to consumers through a lower output price both before and after the tax decrease (Section 3.6.4).

If government wishes to avoid windfall gains this analysis suggests that this type of tax decrease should be imposed with a relatively long information lead.

4.3 Comparative Statics

The long-run equilibrium results for the above examples and other cases are summarized in the following proposition:

-
- 1) Successive information produces shifts in investment and windfall gains at several points of time (Section 3.10).

Proposition 4.1 (Comparative statics):

1. An upward shift in $P_L(L)$ implies
 - i. a higher (lower) K^+ if $(\sigma + \epsilon) > 0 (< 0)$
 - ii. a lower L^+
 - iii. a lower X^+ and a higher P_X^+
2. An upward shift in $F_K'(K, L)$ implies
 - i. a higher (lower) K^+ if

$$\left[P_X F_{LL}'' (1 + \epsilon) + \frac{P_X F_L' F_L'}{X} - P_L' \left(\frac{K F_K'}{X} + \epsilon \right) \right] > 0 (< 0),$$
 - ii. a lower (higher) L^+ if

$$[1 + \sigma + \epsilon + (F_L' (r + \delta) \delta P_I' K / F_{KL}'' P_X X)] > 0 (< 0)$$
 - iii. a higher X^+ and a lower P_X^+
3. An upward shift in $F_L'(K, L)$ implies
 - i. a lower (higher) K^+ if

$$[(1 + \sigma + \epsilon + (F_K' P_L' L / F_{KL}'' P_X X))] > 0 (< 0)$$
 - ii. a higher (lower) L^+ if

$$\left[P_X F_{KK}'' (1 + \epsilon) + \frac{P_X F_K' F_K'}{X} - (r + \delta) \delta P_I' \left(\frac{F_L' L}{X} + \epsilon \right) \right] > 0 (< 0)$$
 - iii. a higher X^+ and a lower P_X^+ .
4. An upward shift in r or $P_I(I)$ implies
 - i. a lower K^+
 - ii. a higher (lower) L^+ if $(\sigma + \epsilon) > 0 (< 0)$
 - iii. a lower X^+ and a higher P_X^+ .

Proof in Appendix p. 169.

4.4 Shift in the Supply of Investment Goods

4.4.1 Adjustment Paths

An upward shift in the supply curve of investment goods produces a downward shift in the $I=0$ -curve. This leads to a lower long-run equilibrium capital stock. This is shown in Figure 4:6 and proved in Prop. 4:1.

Information on a future upward shift in $P_I(I)$ induces an upward shift in investment and a windfall capital gain at time t_1 . This is illustrated in Figure 4:6.a. After time t_1 , the path follows a northeast movement with increasing capital and investment up to time t_s .

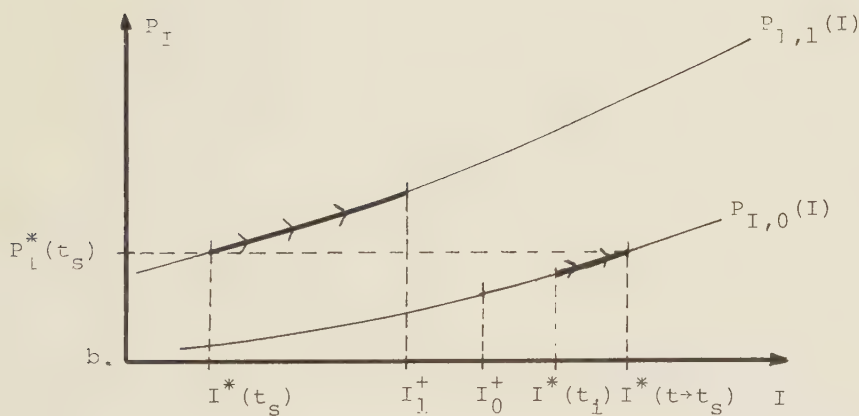
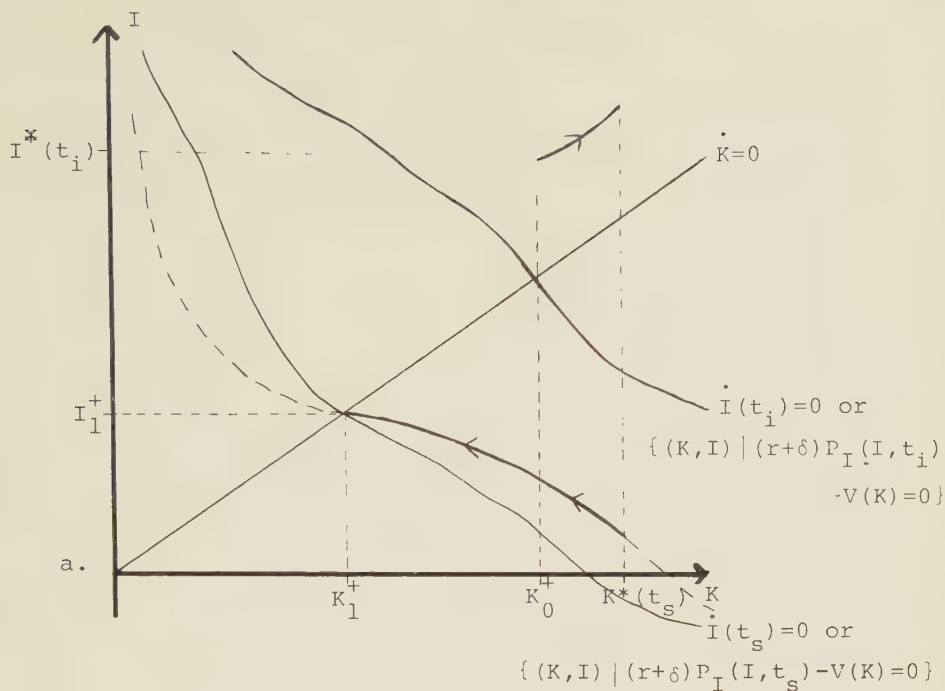


Figure 4:6. Adjustment to an upward shift in $P_I(I)$.

As an intuitive explanation, firms increase investment to hedge versus the future high price of investment goods caused by the high future supply curve for investment goods. (Figure 4:6.b. A formal proof is given in Prop. 4.2 below).

At the time of the shift in $P_I(I)$ investment shifts downwards to a point on the broken curve in Figure 4.6.a, showing all points that lead to the new stationary state. The path then follows the broken curve towards the new long-run equilibrium, as illustrated in Figure 4:6.a. The adjustment path for investment and the price of investment goods is also illustrated in Figure 4:6.b. Note that the downward shift in investment at time t_s compensates for the upward shift in the supply curve, such that investment price is continuous at time t_s . This is illustrated in Figure 4:6.b and 4:7.c. This, at first sight, surprising implication is explained by noting that an expected shift in the price of investment goods is inconsistent with equilibrium, since it would then be profitable to increase capital holdings immediately before the time of the shift.

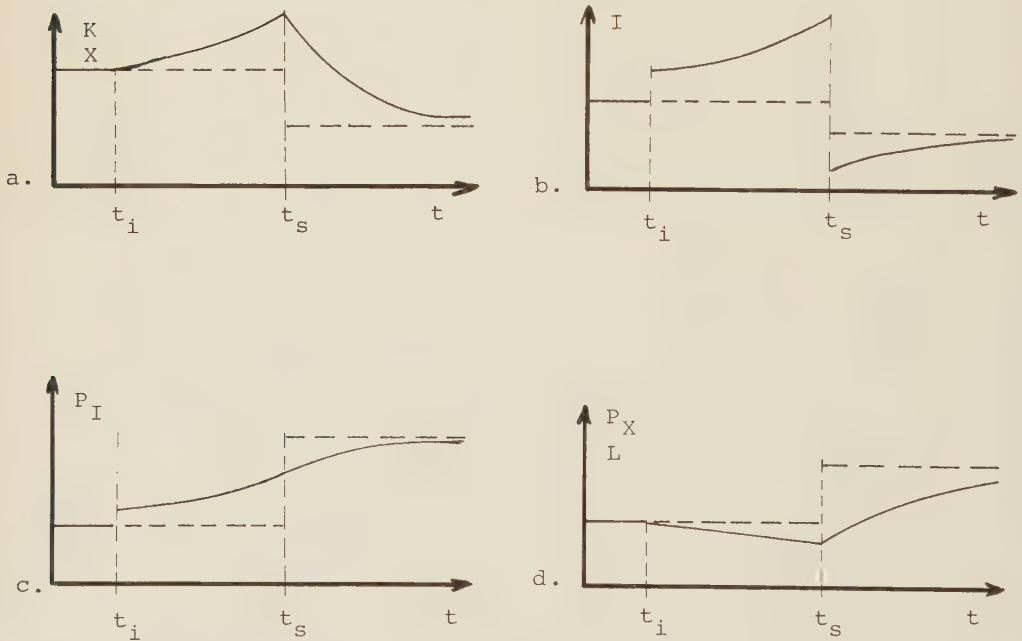


Figure 4:7. Adjustment to an upward shift in $P_I(I)$.

In contrast to other exogenous factors, an expected shift in $P_I(I)$ leads to capital adjustment away from the new stationary state capital before time t_s . By implication, X , P_X , L also move away from their new long-run equilibrium values before time t_s . This is shown in Figure 4:7.a and d.

For the case with an unexpected shift in $P_I(I)$, investment instead shifts downwards to the broken curve in Figure 4:6.a at time $t_i (=t_s)$, with a resulting monotonic decrease in capital towards the new stationary state value. An unexpected basic shift in the supply curve for investment goods therefore decreases investment in accordance with the comparative statics result, (Prop. 4.1). An expected shift increases investment before the shift.

4.4.2 An Application

The foregoing principles may be applied to the effects of, say, an expected tax increase on new cars or, equivalently, an increase in minimum standards for emission, mileage and safety. Such shifts are often considered and implemented in many countries. Our model of adjustment suggests that a tax increase provides wind-fall capital gains to car owners when information emerges. In addition, the sales of new cars shifts upwards at time t_i and increases up to the time of the shift (Figure 4:6.a, 4:7.b). This results in a larger stock of cars before and for some time after the shift (Figure 4:6.a, 4:7.a). This larger stock means a higher volume, X , and a lower price for car driving before and for some time after the tax increase (Figure 4:7.a and d). Equilibrium requires that the price of investment goods changes gradually. The shift in the supply curve, therefore, causes a fall in car sales at time t_s , exactly

offsetting the tendency to price increases (Figure 4:6.b).¹⁾ After t_s car sales are restored successively and converge to the new lower long-run equilibrium (Figure 4:6).

The stock of cars and, consequently, the volume and price of car driving eventually pass their original long-run equilibrium levels and converge towards their new long-run equilibrium values (Figure 4:7.a and d). An expected tax increase is, therefore, predicted to result in a higher stock of cars and a higher volume of car driving before and for some time after the tax increase.

4.4.3 Shifts in the Supply of Investment Goods in the General Case

For the general case with any type of expected shift in the supply curve for investment goods and for any original expected path for the exogenous factors we have the following proposition:

Proposition 4.2: An expected upward shift in the supply curve for investment goods implies:

- i) strictly higher capital for $t \in]t_i, t_s]$
- ii) strictly higher investment for $t \in [t_i, t_s[$
- iii) strictly positive windfall capital gain at time t_i .

Proof in Appendix p. 171.

In the general case firms hedge against an expected upward shift in $P_I(I)$ through higher investment before the shift. No conclusions can be drawn in this case, however, about the effects on investment and capital after time t_s or about the effects of an unexpected

1) Relatively large changes in list prices might be combined with decreased discounts before and increased discounts after the shift, thereby causing a relatively smooth price path.

shift. This is clear since the upward shift in $P_I(I)$ may involve a small shift shortly after time t_s and a large shift at later points of time, in which case the investment stimulating effect of the later shift may be strong enough to increase investment for some time after t_s also. In contrast the basic permanent shift analyzed in 4.4.1 generates lower investment after time t_s , as shown in Figure 4:6.a.

5. ADJUSTMENTS TO TEMPORARY SHIFTS

5.1 Introduction

This chapter examines the effects of introducing basic temporary shifts in exogenous factors into an original stationary state.

5.2 Temporary Shift in Demand

5.2.1 Adjustment Paths for Capital and Investment

Consider the basic temporary demand shift starting at time t_s and ending at time t_e , illustrated in Figure 5:1.a.

Information on the temporary shift results in an upward shift in investment at time t_i (Prop. 3:3). Equations (3.3) and (3.4) produce a subsequent northeast movement up to time t_s , in accordance with the arrows in Figure 5:1.b. The shift in the $I=0$ -curve then produces an investment peak and a southeast movement from that point in time. Since the adjustment path reaches the broken curve smoothly at time t_e (Lemma 3 and Prop. 3.2), the adjustment path must cross the $K=0$ -line before time t_e , with a peak for capital and a southwest movement from the point of intersection at, say, time t_a . The path reaches the broken curve at time t_e . The shift in the $I=0$ -curve then produces an investment minimum and, thereafter, a northwest movement takes place along the broken curve. The adjustment paths for capital and investment are also shown in Figure 5:1.c-d.

As before, the first step in the analysis of adjustment which determines capital and investment paths in Figure 5:1, is applicable not only to a temporary shift in $P_X(X)$ but also to temporary shifts in $P_L(L)$, $F'_K(K,L)$, $F'_L(K,L)$ and r .

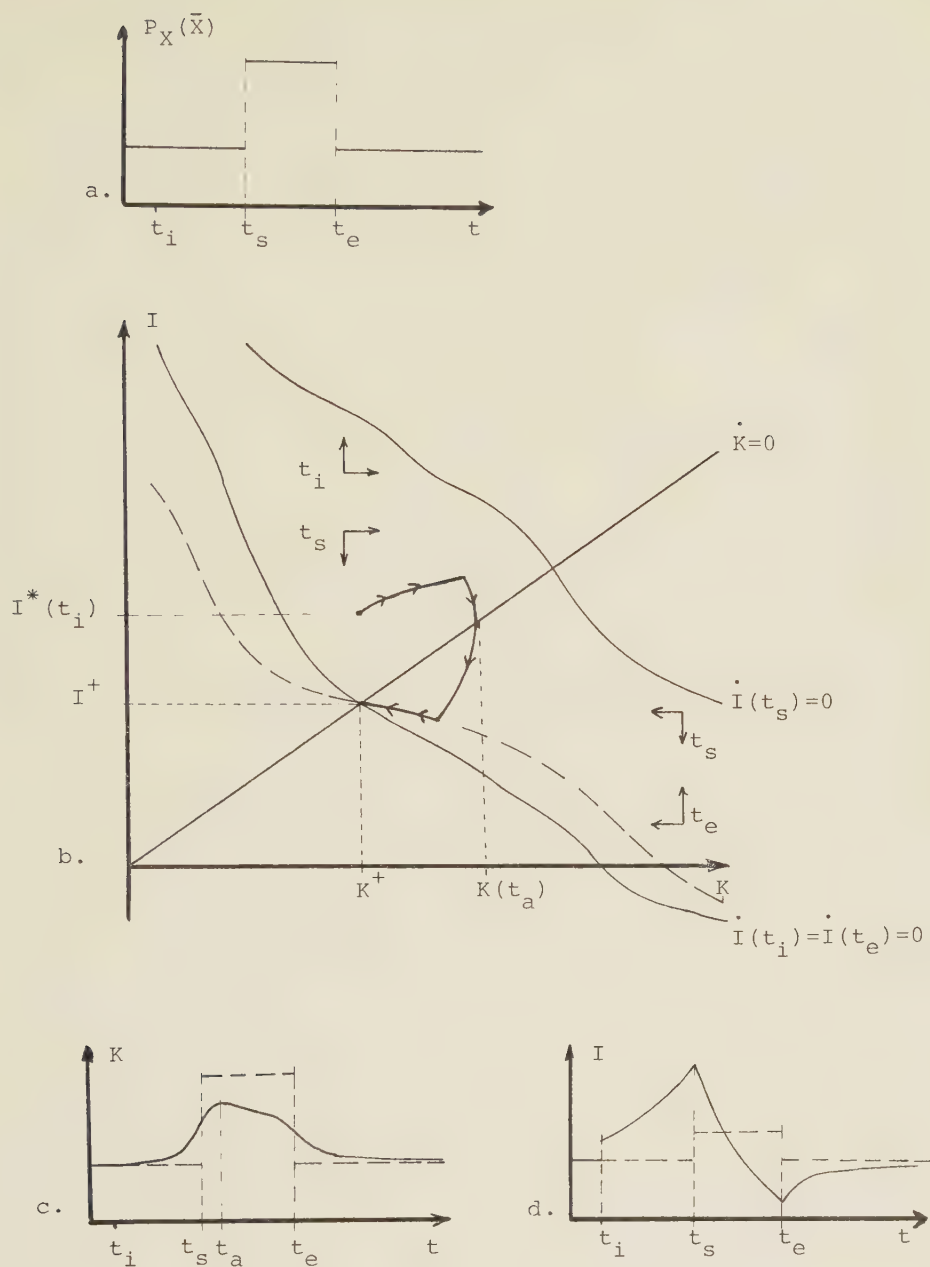


Figure 5:1. Adjustments to temporary shifts in $P_X(X)$, $P_L(L)$, $F'_K(K,L)$, $F'_L(K,L)$ and r .

5.2.2 Adjustment Paths for Output, Output Price and Labor

The shifts in the studied exogenous factors again give different results for the adjustment paths for X , P_X and L . Since the results are quite analogous to those for permanent shifts, the analysis is restricted to the case with a temporary demand shift. This case is illustrated in Figure 5:2.

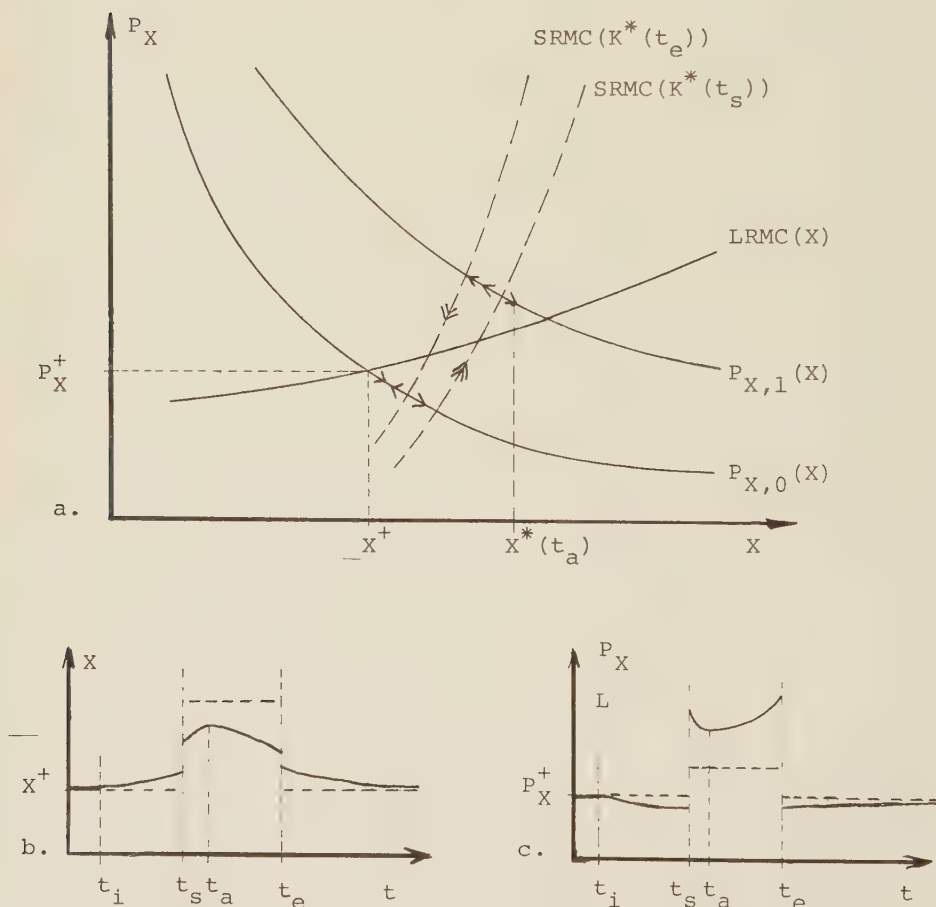


Figure 5:2. Adjustment to a temporary shift in $P_X(X)$.

The increasing capital stock causes a southeast movement along $P_{X,0}(X)$ in Figure 5:2.a up to time t_s (Lemma 2). At t_s the path shifts along the short-run marginal

cost curve for $K^*(t_s)$ to a point on the new demand curve. The continued increase in capital results in a southeast movement along this curve up to time for maximum capital, t_a . Thereafter, the decreasing capital stock leads to a northwest movement along $P_{X,1}(X)$ until time t_e . Output and output price shift along the short-run marginal cost curve for $K^*(t_e)$ to a point on the original demand curve. This is followed by a northwest movement along this curve towards the new and original long-run equilibrium position. These adjustment paths and the L-path for a dominating substitution effect are also shown in Figure 5:2.b-c.¹⁾

A temporary shift, therefore, produces an adjustment of K and X towards the temporary stationary state levels before and for some time after t_s . At some time after t_s but before t_e K and X reach extreme values, and start to adjust back towards their new and original long-run equilibrium levels.

5.2.3 Determinants of the Degree of Temporary Capital Adjustment

As is clear from Section 3.7, a higher δ , a higher r , a higher absolute V -elasticity and a lower P_I -elasticity result in larger capital adjustment towards its temporary stationary state level. This is illustrated by the shift from paths 1 to paths 2 in Figure 5:3.

On an intuitive level, these changes generate more short-sighted investment and, consequently, relatively strong capital adjustment to temporary shifts. A relatively flat supply curve for investment goods, for example, produces more volatile investment and a high degree of capital adjustment to the temporary shift.²⁾

-
- 1) $K^*(t_e)$ could alternatively exceed $K^*(t_s)$, in case the short-run marginal cost curves in Figure 5:2.a change places.
 - 2) Changes in the absolute V -elasticity and the P_I -elasticity also affect the degree of investment fluctuations by affecting the size of the change in stationary state capital and investment. This is further analyzed in Section 8.3.

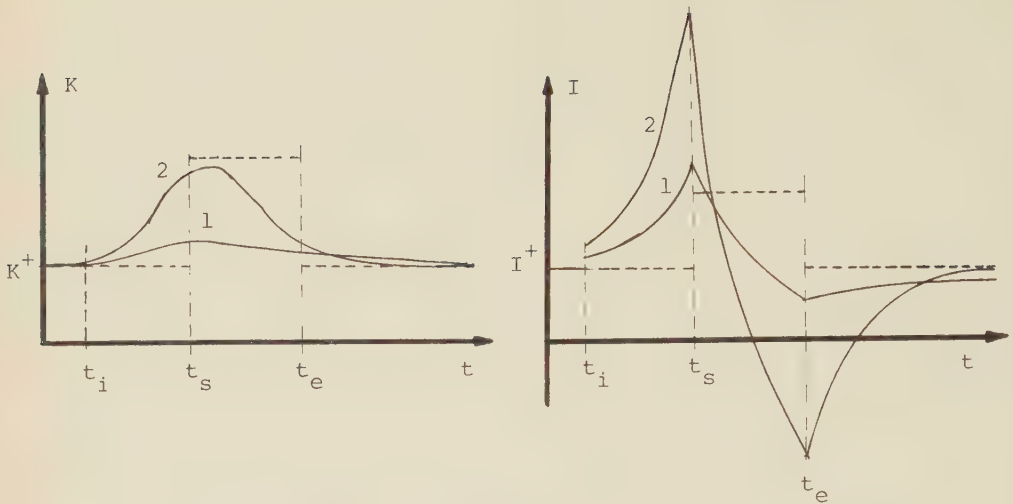


Figure 5:3. Effects of higher values for δ , r , $|V\text{-elasticity}|$ and lower value for $P_I\text{-elasticity}$.

5.2.4 Effects of a Longer Shift

To analyze the effects of a longer time period for a temporary shift, assume the period is extended from $t_{e,1}$ to $t_{e,2}$. This may be interpreted as an additional temporary shift for the time interval $[t_{e,1}, t_{e,2}[$, with associated higher investment up to time $t_{e,1}$ and a larger capital stock at each point in time. This is illustrated by the shift from paths 1 to paths 2 in Figure 5:4 (Prop. 3:3). The higher capital for the longer temporary shift implies lower investment after time $t_{e,2}$, as shown in Figure 5:4.b (Prop. 3.3).

A permanent shift is the limiting case when the end point of time approach infinity. A permanent shift, therefore, generates higher investment than a temporary shift at least up to the end for the temporary shift ($t_{e,1}$), and a larger capital stock at each point in time.

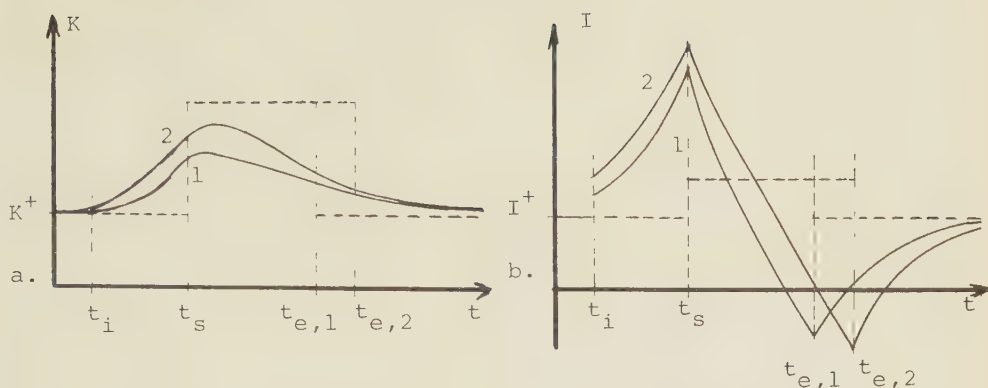


Figure 5:4. Effects of a longer period for a temporary shift.

5.3 Temporary Shift in the Supply of Investment Goods

We analyze the effect of a temporary shift in $P_I(I)$ with the example of a temporary investment subsidy, which is intended to stimulate investment during a trough in the business cycle. Such subsidies are often used in Sweden as part of policies intended to stabilize aggregate demand.

The effects of such a subsidy, illustrated in Figure 5:5.b, is to cause a temporary shift in the $\dot{I}=0$ -curve in Figure 5:5.a.

Information on the subsidy induces a downward jump in investment at time t_i , i.e. investment is postponed in anticipation of the subsidy (Prop. 4.2). Figure 5:5.a illustrates the resulting southwest movement with decreasing capital and investment up to time t_s . The introduction of the subsidy then gives rise to an upward shift in investment. This shift compensates for the introduction of the subsidy resulting in a continuous price

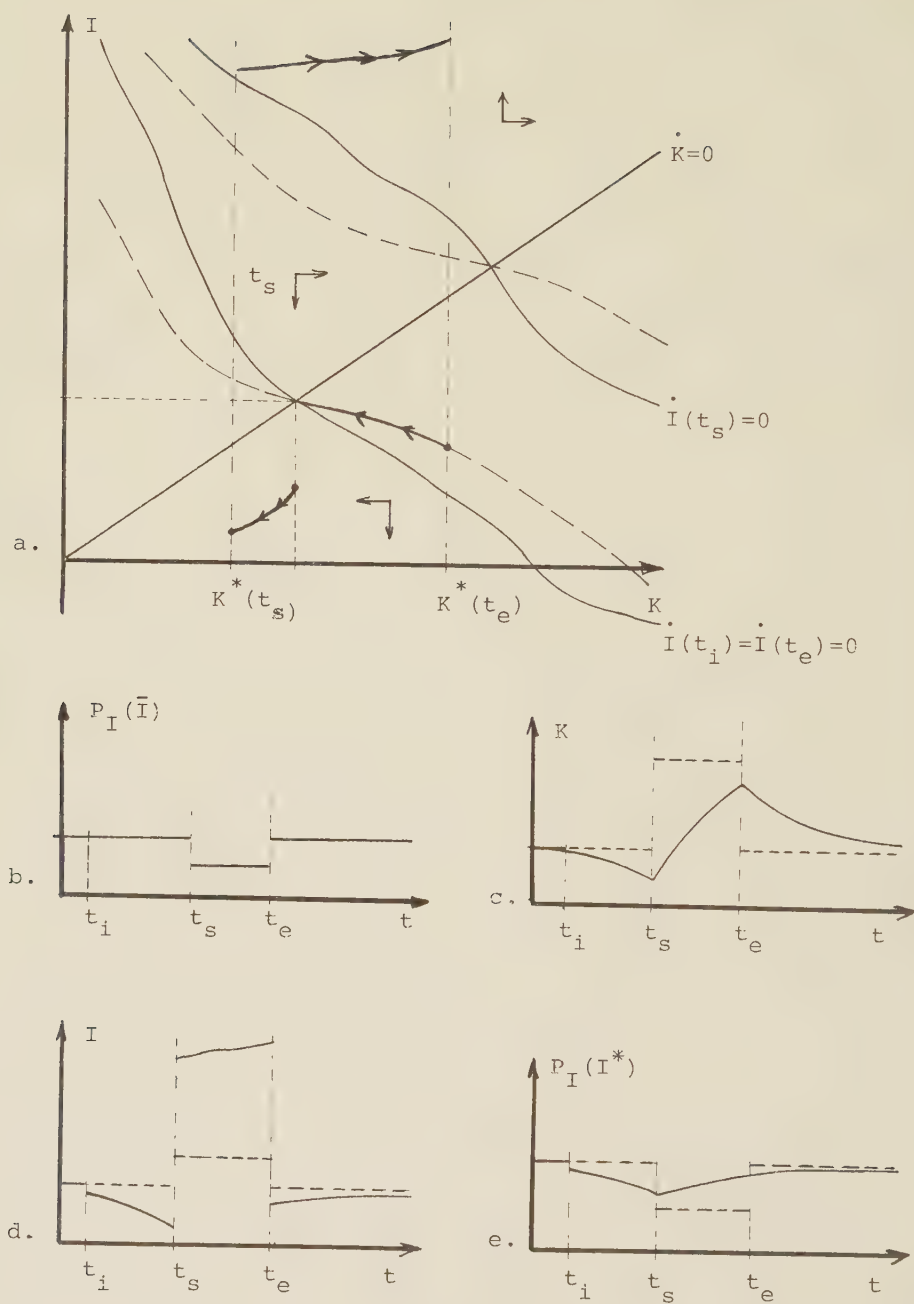


Figure 5:5. Adjustment to a temporary shift in $P_I(I)$.

path for investment goods in Figure 5:5.e. Investment shifts to a point above the upper broken curve in Figure 5:5.a. This is clear since a constant future supply curve produces a shift to the broken curve and the expected upward shift in $P_I(I)$ at time t_e increases investment as compared to this case (Prop. 4.2). In Figure 5:5.a investment also exceeds the $I(t_s)=0$ -curve yielding a northeast movement with increasing investment during the subsidy period. Alternatively, investment may shift to a point between the upper broken and the upper solid curves at time t_s , involving a southeast movement with decreasing investment for part or all of the subsidy period. In both cases capital increases up to time t_e . The elimination of the subsidy causes a downward shift in investment at this point of time to a point on the lower broken curve followed by an adjustment with decreasing capital and increasing investment to the long-run equilibrium position. The adjustment paths are also shown in Figure 5:5.c-e.

The predictions of dampened investment before the subsidy period and stimulated investment during the period (Figure 5:5.d) suggest that traditional decision methods with lags for observation, discussion, decision and implementation may involve a destabilization of aggregate demand! This is illustrated in Figure 5:6 for a simple four year business cycle and a reasonable total lag.

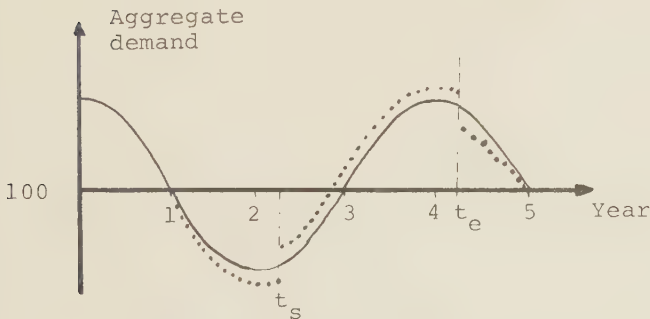


Figure 5:6. Effect of a poorly timed temporary investment subsidy.

To be successful, intervention for the purpose of stabilization requires a highly forward-looking policy. The subsidy should, indeed, be discussed and agreed upon when current and slightly lagged observations suggest high aggregate demand. If the total lag is ten months the properly timed investment subsidy in Figure 5:7 requires actions when latest available data shows a peak in aggregate demand!¹⁾

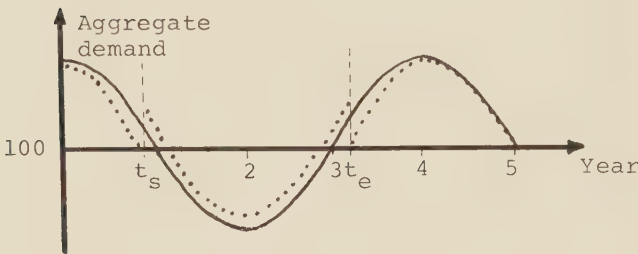


Figure 5:7. Effect of a properly timed temporary investment subsidy.

1) For a further analysis of the timing of stabilization policy see Phillips (1954, 1957) and Thalberg (1971b).

6. THEORY OF INVESTMENT FOR THE INDUSTRY

6.1 Introduction

The model developed in Chapter 2 determines investment plans for each firm and, as an aggregate over firms, for the industry, given any expected time paths for the exogenous factors. More specifically for any expected time paths,

$$\{P_X(X, t), P_L(L, t), P_I(I, t), F'_K(K, L, t), F'_L(K, L, t), r(t) \text{ for } t \geq t_p\},$$

each firm calculates expected equilibrium price paths:

$$\{P_X^*(t), P_L^*(t), P_I^*(t) \text{ for } t \geq t_p\}$$

From these expected prices the firms determine their optimal quantity plans

$$\{X^{i,*}(t), I^{i,*}(t), L^{i,*}(t) \text{ for } t \geq t_p\} \text{ for } i=1, 2, \dots$$

The implied aggregate industry level paths are

$$\{X^*(t), I^*(t), L^*(X) \text{ for } t \geq t_p\}$$

As long as no new information arises the development follows these plans. Given that firms know current prices, the solution for the current point in time determines not only planned but also actual investment. Our model, therefore, also provides a theory for determination of actual investment for the firm and the industry.

This chapter uses this theory to examine the determination of investment for the industry for any expected paths for the exogenous factors.

6.2 Comparative Dynamics of Investment Determination

If the solution to the industry level optimization problem (2.6) is written as a function of the expected time paths, the determination of actual investment for the industry might be expressed as:

$$(6.1) \quad I^*(t_p) = \Psi \left[\{P_X(X, t), P_L(L, t), P_I(I, t), F'_K(K, L, t), F'_L(K, L, t), r(t) \text{ for } t \geq t_p\}, K(t_p) \right]$$

The comparative dynamics results for the effects of any type of upward shift in Propositions 3.3 and 4.2 lead to the following proposition:

Proposition 6:1: (Comparative Dynamics) Actual investment for the industry is:

1. strictly increasing in upward shifts in the demand function.
2. strictly increasing (decreasing) in upward shifts in the supply function for labor if the substitution effect dominates (does not dominate) the output effect, i.e. if $(\sigma + \varepsilon) > 0 (< 0)$.
3. strictly decreasing in momentary temporary upward shift in current supply function for investment goods.
4. strictly increasing in upward shifts in the future supply function for investment goods.
5. strictly increasing (decreasing) in upward shifts in the marginal production function for capital if
$$\left[P_X F''_{LL} (1 + \varepsilon) + \frac{P_X F'_L F'_L}{X} - P'_L \left(\frac{KF'_K}{X} + \varepsilon \right) \right] > 0 (< 0).$$
6. strictly decreasing (increasing) in upward shifts in the marginal production function for labor if
$$[1 + \sigma + \varepsilon + (F'_K P'_L / F''_{KL} P_X X)] > 0 (< 0).$$
7. strictly decreasing in upward shifts in the interest rate.
8. strictly decreasing in inherited capital.

Proof in Appendix p. 172.

These results are illustrated in (6.2) for the first of the cases considered in 2, 5 and 6.

(6.2)

$$I(t_p^*) = \Psi [P_X^+(X, t), P_L^+(L, t), F_K^{'+}(K, L, t), F_L^{'+}(K, L, t), r^-(t) \text{ for } t \geq t_p, \\ P_I^-(I, t_p), P_I^+(I, t) \text{ for } t > t_p, K(t_p)]$$

These comparative dynamics results are in accordance with earlier comparative statics results, (Prop. 3.1 and 4.1), except for the case with an upward shift in the future supply curve for investment goods where investment is stimulated before the shift. Moreover, the effects of a shift in inherited capital cannot be analyzed by comparative statics.

6.3 Investment as Optimal Capital Adjustment

6.3.1 Introduction

This section analyzes the determination of the optimal capital path as an adjustment towards a moving target for any expected time paths for the exogenous factors. Since capital path is determined by the paths $\{V(K, t), r(t), P_I(I, t) \text{ for } t \geq t_p\}$ only in Equations (2.15) - (2.17), the analysis is based on any expected paths for these factors. The marginal revenue product function summarizes all relevant information in $P_X(X, t), P_L(L, t), F_K^I(K, L, t)$ and $F_L^I(K, L, t)$.

6.3.2 Constant Path for Supply of Investment Goods and Interest

For simplicity, the analysis starts with the case where $P_I(I)$ and r are expected to remain constant over time. The $V(K)$ -function might have any time path, as illustrated in Figure 6:1.a.

Stationary state capital or target capital at time t is the capital where capital and investment are constant. From (3.1) and (3.2) target capital is:

$$(6.3) \quad K^+(t) \triangleq (K | V(K, t) = (r(t_p) + \delta) P_I(\delta K, t_p))$$

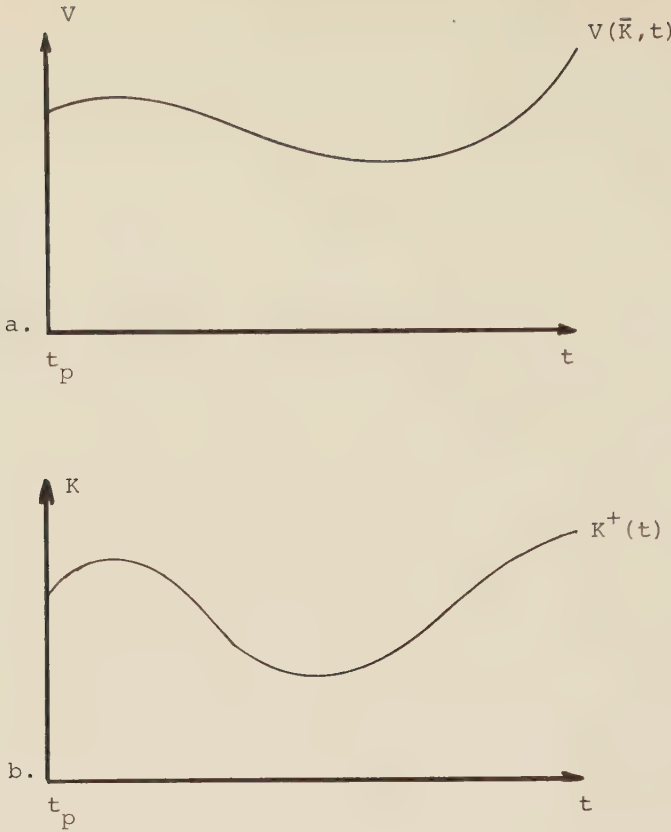


Figure 6:1. Time paths for the marginal revenue product function and target capital.

Target capital is determined by the intersection between the $K=0$ -line and the $\dot{I}=0$ -curve in the phase diagram in Figure 3:3 above. The changing $V(K)$ -function illustrated in Figure 6:1.a produces a changing $\dot{I}=0$ -curve and, consequently, a changing target capital. This is illustrated in Figure 6:1.b. If $V(K)$ and $P_I(I)$ are linear functions and the $V(K)$ -function has a constant slope over time, Equation (6.3) reveals that $K^+(t)$ is a linear function of $V(\bar{K}, t)$. This involves identical shapes for the paths in Figure 6:1. Otherwise, the paths might have different shapes, as is the case in Figure 6:1.

Our investment theory for the industry gives the

following picture of investment determination. On the basis of different types of information, firms formulate expectations for the market's future development. This is illustrated by the expected path in Figure 6:1.a. Interpreted as an optimization for the industry, this expected path is then used to determine the industry's optimal capital path. More specifically, inherited capital, $K(t_p)$, is compared to target capital for future points of time. Even if inherited capital happens to be equal to target capital at time t_p , expected future shifts in exogenous factors induce capital adjustment at time t_p (Prop. 3.3).

The planned capital path shows some gravitation towards target capital at each future point of time (Prop. 3.3), with a tendency to strongest attraction towards target capital in the near future (Section 6.4). The optimal capital path is determined, therefore, as a partial adjustment of inherited capital towards a moving target. Optimal capital adjustment at a given point of time depends on the entire future path for target capital. This gives a smoothened optimal capital path, as illustrated by the dotted curve in Figure 6:2.a.

By way of illustration, consider a particle that passes through a field with many points of gravitation, shown by the solid curve in Figure 6:2.a. The influence of each point ends when the particle passes. The determination of the path for such a particle moving with a constant speed in the horizontal direction is quite analogous to the determination of the optimal capital paths, starting at $K(t_p)$ in Figure 6:2.a.

The equilibrium capital path determines the equilibrium investment path:

$$I^*(t) = \delta K^*(t) + \dot{K}^*(t)$$

This is illustrated by the dotted curve in Figure 6:2.b.

This interpretation suggests that actual investment, $I^*(t_p)$, is determined as the optimal adjustment of inherited capital towards the time path for target capital.

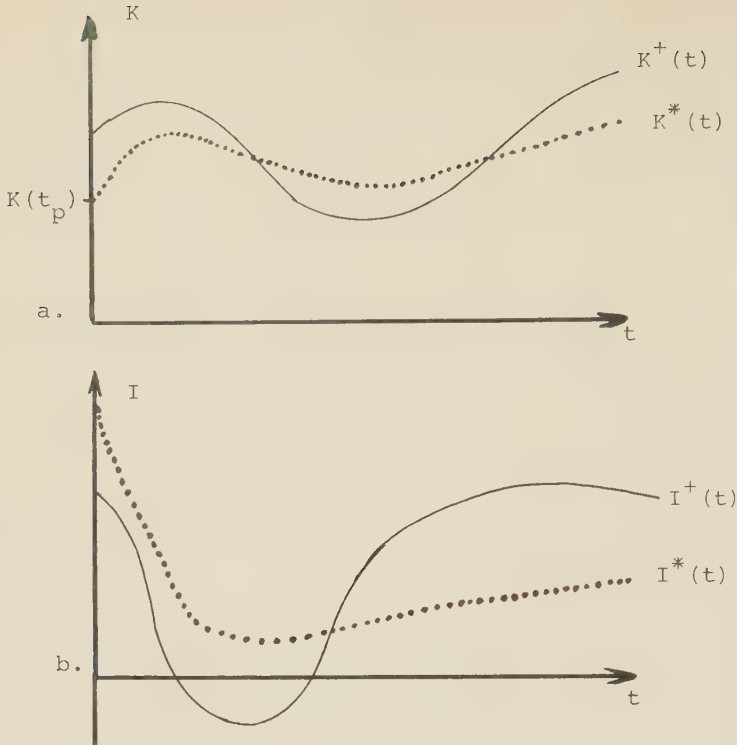


Figure 6:2. Capital adjustment towards a moving target.

tal. Since optimal capital adjustment depends on the entire path for target capital, investment depends on the entire path for the exogenous factors. Analogously, planned investment for a future point in time is determined as the optimal adjustment of future capital towards the moving target.

Alternatively the optimal investment plan may be viewed as a smoothening of target investment, $I^+(t) \triangleq \dot{K}^+(t) + \delta K^+(t)$, and the infinite initial investment corresponding to the jump $K^+(t_p) - K(t_p)$. This is illustrated by the dotted curve as compared to the solid curve in Figure 6:2.b.

6.3.3 Constant Target

In the special case where all exogenous factors are expected to remain constant after a finite point of time, t_f , target capital is constant thereafter. The optimal capital path then converges to a constant target after time t_f , as illustrated in Figure 6:3 (Prop. 3.2).

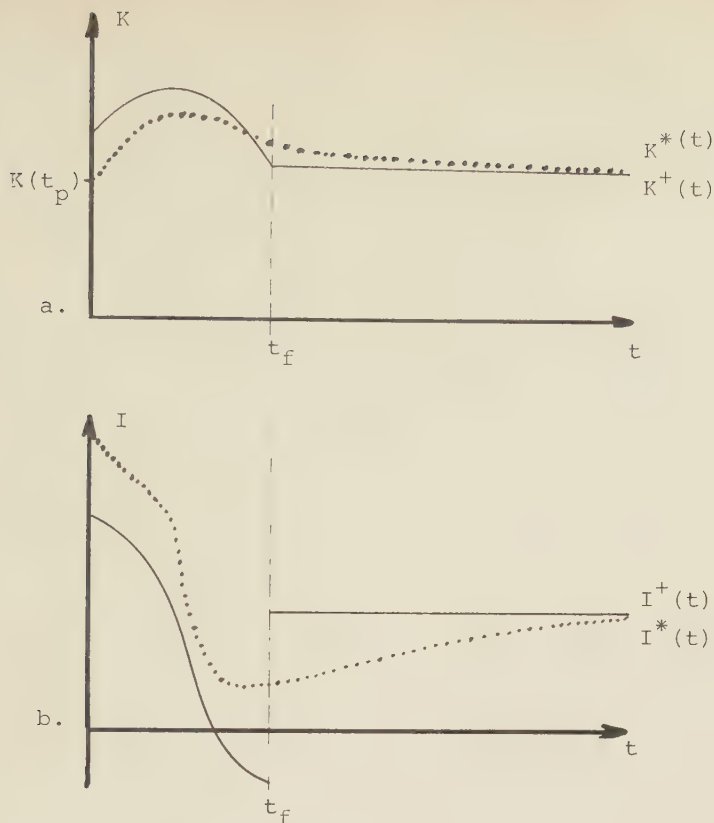


Figure 6:3. Capital adjustment towards a constant target.

The moving-target-chasing model then reduces to a simple constant-target-chasing model after time t_f .

6.3.4 Any Path for Supply of Investment Goods and Interest

For the general case with any expected time path for $P_I(I)$ and r , target capital producing constant capital and investment is:

$$(6.4) \quad K^+(t) \triangleq (K|V(K,t) = (r(t) + \delta)P_I(\delta K, t) - \frac{\partial P_I}{\partial t}(\delta K, t)).$$

In comparison with the case with a constant supply curve for investment goods, a supply curve increasing over time means a higher target capital during at least a part of the increase. This is illustrated by the broken curves in relation to the solid curves in Figure 6:4.a-b. The result is explained by the desire to have

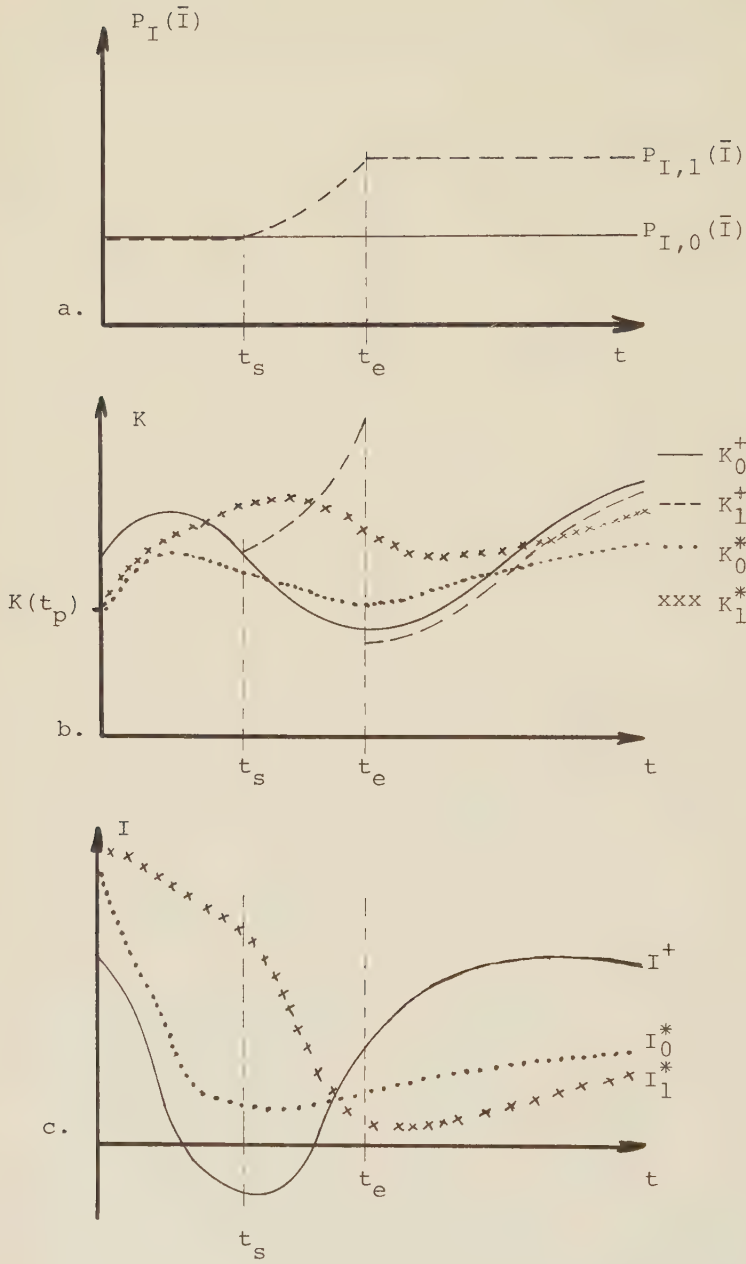


Figure 6:4. A changing supply curve for investment goods.

a high capital when the increasing supply curve tends to involve capital gains on existing capital.

The optimal capital path is again determined as the optimal adjustment towards a moving target, producing higher optimal capital before the increase in relation to the case with a constant supply curve. This is demonstrated by the crossed path relative to the dotted path in Figure 6:4.b, (Prop. 4:2). This explains the earlier result that actual investment, $I^*(t_p)$, increases with future upward shifts in the supply curve of investment goods, (Prop. 4:2).¹⁾

6.4 Relative Importance of Near and Distant Future

It has been demonstrated that actual and planned investment are determined as the optimal adjustment of inherited capital towards a moving target capital. This section examines the conditions under which $I^*(t_p)$ is determined mainly by target capital in the near future. This is then contrasted with the circumstances under which distant target capital exerts a significant impact on investment. We are, therefore, concerned to examine short- and long-sighted investment.

We begin by considering the effects on $I(t_p^*)$ of a given temporary shift in the V-function, when the shift starts twelve months from now and contrast this with the effects

- 1) This section is a simplification in that the analysis only considers how the K^* -path is determined for a given K_+ -path. A local upward shift in the V-function around K^* might, however, increase investment without affecting $V(K_+)$ and, consequently, without affecting K_+ . The path for target capital does, therefore, not give a complete picture of the determination of capital and investment except for the case with linear $P_I(I)$ - and $V(K)$ -functions with constant slopes over time.

of a shift starting immediately. For shifts introduced into an original stationary state, Proposition 3:4.iv states the reasonable result that the later is the shift, the smaller is the effect on investment. This means that near target capital exerts a stronger influence on investment than distant target capital.

As analyzed in Section 3.7, a high rate of depreciation leads to fast and concentrated capital adjustment. The shift twelve months ahead has then a much weaker effect on investment than an immediate shift, i.e. a high δ gives rise to shortsighted investment (*ceteris paribus*). The analysis in Section 6.3 suggests the interpretation that a high δ causes a strong gravitation towards current target capital. Optimal capital then follows target capital closely as illustrated by the K_1^* -path in Figure 6:5. With a low δ there is only

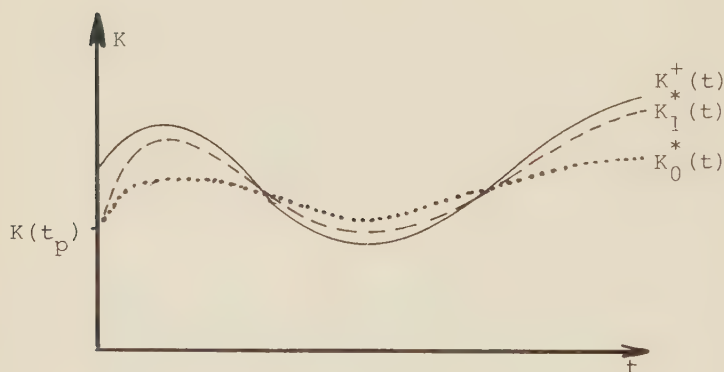


Figure 6:5. Effects of a higher δ , a higher r , a higher $|V\text{-elasticity}|$ and a lower $P_I\text{-elasticity}$.

weak gravitation towards current target capital, involving a smooth path as shown by the K_0^* -path in Figure 6:5. The implication of high δ is, therefore, that capital adjusts mainly towards target capital in the near future, i.e. investment is shortsighted. In the limiting case where δ approaches infinity both inputs are non-durable, producing instantaneous adjustment and $K^*(t) = K^+(t)$. Since $I^*(t) = K^*(t)$ investment is determined

by current values for the exogenous factors only, i.e. investment is extremely shortsighted.

Analogously, a high interest rate, a high absolute V-elasticity and a low P_I -elasticity produce fast and concentrated capital adjustment as illustrated by path K_1^* in Figure 6:5. Capital then follows target capital closely. Investment is shortsighted, being dominated by target capital in the near future.

In the limiting case when P_I -elasticity approaches zero, $K^*(t) = K^+(t)$. Investment is then determined by current values for exogenous factors and current changes in the exogenous price of investment goods only (Section 2.7). Again, investment is extremely shortsighted.

To summarize, a higher rate of depreciation in basic capital, δ_b , a higher long-run growth rate, g_L , a higher r , a higher absolute V-elasticity and a lower P_I -elasticity lead to more shortsighted investment. The numerical results in Section 3.7 indicate that the degree of shortsightedness is especially sensitive to the depreciation rate.

6.5 Microeconomic Basis for the Partial Adjustment Model

6.5.1 Introduction

This section demonstrates that, under some simplifying assumptions, investment determination for the industry may be approximated by the partial adjustment model:

$$(6.5) \quad \dot{K} = \eta(K^+ - K)$$

K^+ is target capital determined by the expected paths for the exogenous factors and η is the speed of adjust-

ment.¹⁾

6.5.2 Derivation of the Partial Adjustment Model

We first examine the case in which all exogenous factors are expected to remain constant over time. An approximation of Equation (3.4) by a linear function in I and K around the stationary state values and the equation determining net investment may be written:

$$(6.6) \begin{pmatrix} \dot{I} \\ \dot{K} \end{pmatrix} = \begin{pmatrix} (r+\delta) & -v'(K^+)/P_I^+(I^+) \\ 1 & -\delta \end{pmatrix} \begin{pmatrix} I-I^+ \\ K-K^+ \end{pmatrix}$$

We then have:²⁾

$$(K-K^+) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t},$$

$$\text{Where } \lambda_1 (\lambda_2) = \frac{r}{2} (\pm) \sqrt{\left(\frac{r}{2}\right)^2 + (r+\delta)\delta - (v'/P_I^+)^2}$$

Since λ_1 is strictly positive, convergence (Prop. 3.2) implies that c_1 equals zero. It follows that:

$$(6.7) \quad \dot{K} = \lambda_2 c_2 e^{\lambda_2 t} = -\lambda_2 (K^+ - K), \text{ i.e.}$$

(6.8)

$$\dot{K} = \left(\sqrt{\left(\frac{r}{2}\right)^2 + (r+\delta)\delta (1 + (|v^+ - e|)/P_I^+ - e|))} - \frac{r}{2} \right) (K^+ - K)$$

$$\text{where } v^+ - e| \triangleq \frac{\partial \ln v(K^+)}{\partial \ln K} \text{ and } P_I^+ - e| \triangleq \frac{\partial \ln P_I(I^+)}{\partial \ln I}.$$

Equation (6.8) provides the desired microeconomic basis for the partial adjustment model (6.5) when all exogenous factors are expected to be constant over time.

1) The partial adjustment model, also called The Flexible Accelerator, is developed and used by, for example, Chenery (1952), Griliches (1960), Koyck (1954) and Smithies (1957). Eisner and Strotz (1963) and Lucas (1967a) provide a microeconomic basis for the partial adjustment model for the firm in a competitive market with stationary price expectations and internal adjustment costs.

2) See Intriligator (1971, Appendix A).

We next turn to the case where r and $P_I(I)$ are expected to be constant over time and $V(K)$ is approximated by a linear function, for which the position but not the slope, changes over time. If the $P_I(I)$ -function is also approximated by a linear function, the partial adjustment model has a solid microeconomic basis also in this case. To see this, note from Equation (2.15) that the effects of parallel V -shifts on investment are additive under these assumptions. The total effects on investment of two shifts are then equal to the sum of the effects of the separate shifts. For the determination of $I^*(t_p)$, any time path for the linear $V(K)$ -function might then be substituted by a constant average V -function which is defined as:

$$(6.9) \quad AV(K, t_p) \triangleq \int_0^{\infty} V(K, t_p + i) w(i) di,$$

$w(i)$ shows the effect on $I^*(t_p)$ of a given momentary temporary shift in $V(K)$ at time $t_p + i$, as a proportion of the effect of the same permanent shift at time t_p . Due to additivity $\int_0^{\infty} w(i) di = 1$.

As an explanation, any $V(K)$ -path might be interpreted as a sequence of momentary temporary shifts. Due to additivity, a given momentary temporary shift i time periods ahead might be substituted by a basic permanent shift of magnitude $w(i)$ times the size of the temporary shift for the determination of $I^*(t_p)$.

The partial adjustment model (6.8) is, therefore, applicable also to the case with any path with parallel $V(K)$ -functions. Target capital, K^+ , is then determined from the average V -function defined in (6.9).

The introduction of an average V -function simplifies the optimization problem at time t_p . It is no longer the problem of chasing a moving target. Instead it becomes the problem of chasing a constant target. Since the average V -function normally changes over time the constant target used to determine investment at time t_p normally differs from the constant target used at

time $t_p + Z$.

A higher δ , a higher r , a higher $|V\text{-el.}|$ and a lower $P_I\text{-el.}$ bring about more shortsighted investment through two effects in the partial adjustment model:

1. Changes in these variables increase the relative importance of a near shift as compared to a distant shift, with a consequent faster decrease in the weights $w(i)$ in (6.9).
2. Such changes determine a higher coefficient for the speed of adjustment in (6.8).

6.5.3 Extensions and Remaining Limitations of the Partial Adjustment Model

Our analysis suggests the following extensions and modifications to the traditional partial adjustment model for the industry:

1. Desired capital is determined from a weighted average of the expected path for the $V(K)$ -function to account for any expected paths for $P_X(X)$, $P_L(L)$, $F'_K(K, L)$, and $F'_L(K, L)$. Higher δ , r , $|V\text{-el.}|$ and lower $P_I\text{-el.}$ result in larger weights for the near future.
2. The partial adjustment model is derived under the assumption that r , $V^+\text{-el.}$ and $P_I^+\text{-el.}$ are expected to be constant over time. These factors may, however, change ex post, in which case the coefficient for the adjustment speed in Equation (6.8) also changes. This suggests a formulation of the partial adjustment model in empirical estimations where the adjustment speed as well as the weights in the average V -function are functions of δ , r , $V^+\text{-el.}$ and $P_I^+\text{-el.}$
3. The microeconomic basis for the partial adjustment model is derived for growth deflated basic variables, as analyzed in 3.2.3. This suggests that empirical estimations should be made for growth deflated basic variables too. The rate of depreciation, for example, is then the sum of the long-run growth rate and the depreciation rate for basic capital.

The analysis also highlights the following remaining limitations of the partial adjustment model:

1. The assumption of constant expected paths for r and $P_I(I)$ excludes investment based on expected changes in these factors. The analysis in Section 6.5 suggests that expected changes in these factors may be introduced through ad hoc adjustments in target capital. This section does not, however, provide any solid microeconomic basis for such a procedure. Since investment is decreasing in $P_I(I, t_p)$ but increasing in $P_I(I, t > t_p)$ (Prop, 6:1), models based on an average $P_I(I)$ -function might generate highly misleading results.
2. The linear approximation for the $V(K)$ - and $P_I(I)$ -functions and the assumption of a constant expected slope for the $V(K)$ -function may involve large approximation errors. Large variations in investment and an increasing slope of the supply curve for investment goods might give particularly large errors.

Our extended partial adjustment model appears to produce a reasonable approximation to investment determination when the exogenous factors are expected to be fairly constant over time. Investment is then mainly determined as optimal adjustment towards a constant target. The partial adjustment model appears to be a poor approximation, however, when investment is principally determined by expected changes in the exogenous factors, especially when the supply curve for investment is expected to change over time.

6.6 Relation to Keynes' Investment Theory

6.6.1 Introduction

This section demonstrates that our investment theory for the industry may be interpreted as a formalization of Keynes' (1936) investment theory.

6.6.2 Investment Determination in Keynes' Model

Keynes' investment theory, (1936, Ch. 11-12), states

that for a given long-run rate of interest, $r^{lr}(t_p)$, investment is determined such that the sales price of investment goods, P_I , equals the sum of the discounted prospective yields from a marginal investment, $\int_{t_p}^{\infty} Q(I(t_p), s) e^{-r^{lr}(s-t_p)} ds$. ($Q(I(t_p), s)$ is prospective yield at time s .) The former is increasing and the latter decreasing in investment. This determination of investment is illustrated in Figure 6:6.

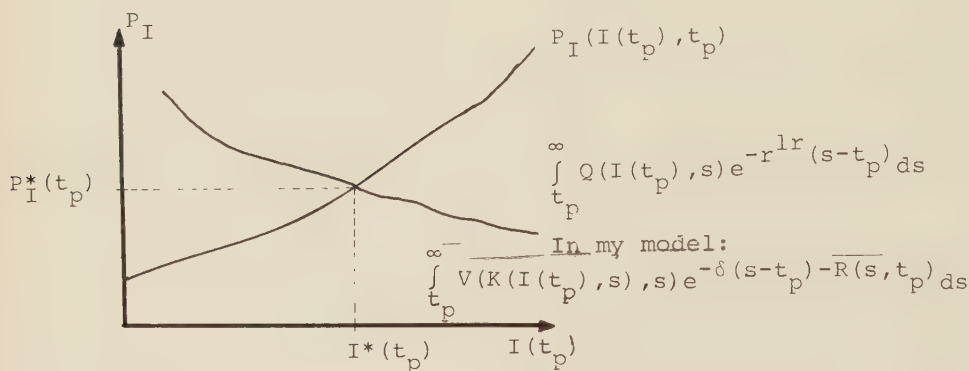


Figure 6:6. Determination of investment.

For any initial investment, $I(t_p)$, our phase equations (3.1)-(3.2) determine the future capital path. Let $K(I(t_p), s)$ denote capital at time s , on the path determined for $I(t_p)$. Our model of investment determination then reduces to:

(6.10)

$$P_I(I(t_p), t_p) = \int_{t_p}^{\infty} V(K(I(t_p), s), s) e^{-\delta(s-t_p) - R(s, t_p)} ds,$$

Thus, we arrive at $I^*(t_p)$, as illustrated in Figure 6:6.

Keynes' prospective yield corresponds, therefore, to the depreciated marginal revenue product for the equi-

librium capital path in our model. His long-run interest rate corresponds to a weighted average of our short-run interest rates.

Keynes' discussion of $P_I(I)$ and $Q(I,s)$ shows striking similarities with our model:

1. As investment is increased "pressure on the facilities for producing that type of capital will cause its supply price to increase" (1936, p 136), i.e. $P_I^1 > 0$.
2. "...prospective yield will fall as the supply of that type of capital is increased" (p 136).
i.e. $\frac{\partial Q}{\partial I} < 0$.
3. "The considerations upon which expectation of prospective yield are based are partly existing facts which we can assume to be known more or less for certain, and partly future events which can only be forecasted with more or less confidence. Amongst the first may be mentioned the existing stock of various types of capital-assets and of capital assets in general and the strength of consumers' demand for goods which require for their efficient production a relatively larger assistance from capital." (This corresponds to $K(t_p), P_X(X, t_p)$ in our model.) "Amongst the latter are future changes in the type and quantity of the stock of capital-assets and in the tastes of the consumer, the strength of effective demand from time to time during the life of the investment under consideration, and the changes in the wage units in terms of money, which may occur during its life" (page 147) (This corresponds to $K^*(t), P_X(X, t)$ and $P_L^*(t)$ for $t > t_p$).

6.6.3 Marginal Efficiency of Investment

If the interest rate is expected to be constant, our investment model (6.2) determines investment as a decreasing function of the interest rate for given expected paths for the other factors. This is illustrated in Figure 6:7.

This curve corresponds to the schedule for the marginal

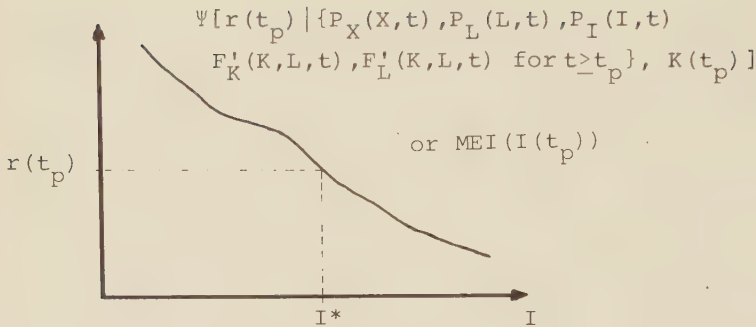


Figure 6:7. Schedule for the marginal efficiency of investment.

efficiency of investment defined as:¹⁾

(6.11)

$$MEI(I(t_p)) \triangleq [r^{rl} | P_I(I(t_p), t_p) = \int_{t_p}^{\infty} Q(I(t_p), s) e^{-r^{lr}(s-t_p)} ds]$$

Keynes stressed that the position of the marginal efficiency of investment-schedule is highly sensitive to the "state of confidence" (Ch. 12).

Accordingly, our industry level investment model states that investment depends on the entire future expected paths for the exogenous factors. In a world with imperfect foresight, including uncertainty on the outcome of

- 1) Keynes called this relation 'the schedule of the marginal efficiency of capital'. Lerner (1944), Arvidsson (1960), Wijkman (1965) and Hamberg (1971, ch. 1) argue convincingly that this relation instead should be called 'the marginal efficiency of investment'. The marginal efficiency of capital is, instead, defined as the marginal efficiency of investment in a stationary state with zero net investment. This corresponds to the determination of the interest rate that is consistent with different stationary state capital in our model; i.e.

$$MEK(K^+) \triangleq r(K^+) \triangleq [V(K^+) / P_I(\delta K^+)] - \delta.$$

different types of random events, expectations on the future might be quite volatile. Our model, therefore, incorporates the common view that information and expectations on the future are crucial determinants of investment. Interpreted in Figure 6:7, any shift in the expected paths for $P_X(X)$, $P_L(L)$, $P_I(I)$, $F'_K(K,L)$ or $F'_L(K,L)$ shifts the marginal efficiency of investment-schedule (Prop. 3.3 and 4.2). This supports Keynes' view that this curve is highly sensitive to the "state of confidence".

6.6.4 Extensions of Keynes' Investment Theory

Our theory of investment for the industry may be interpreted as a formalization of Keynes' investment theory with the following extensions:

1. Our investment theory rests on a solid microeconomic basis. The industry's behavior is derived from the behavior of optimizing firms with rational price expectations. Keynes' model includes no explicit theory of the formation of price expectations.
2. Our investment theory determines investment as a part of a dynamic optimization problem determining the entire future paths for capital and the marginal revenue product of capital. Keynes determined investment in a static optimization model, assuming an exogenous time path for the prospective yield, involving the limitations analyzed in Section 3.9 above.
3. Our model determines investment for any time path for the rate of interest, while Keynes only considered a long-run interest rate.
4. Our investment theory determines investment from a time path for the labor supply function, $P_L(L,t)$, and not from a time path for an exogenous price of labor. Moreover, our model explicitly considers the effects of a time-dependent production function and a time-dependent supply function for investment goods.
5. Unlike Keynes, our investment theory includes a theo-

ry of the determination of the degree of forward-looking investment (Section 6.4).

6.6.5 Classical, Keynes' and Neoclassical Investment Theory

From our neoclassical model we obtain an investment theory that is consistent with Keynes' investment theory including, for example, a negative relationship between investment and the interest rate.

Neoclassical models with exogenous prices, however, involve instantaneous capital adjustment. This results in a negative relationship between capital and the interest rate but not between investment and the interest rate, in contrast to Keynes' investment theory.

The assumption of exogenous prices is adequate for the firm but not for the industry in a competitive market. If, instead, the supply price of investment goods is increasing in the industry's investment, this means successive adjustment to shifts in exogenous factors. A higher interest rate then brings about not only a lower long-run equilibrium capital stock, but also lower investment as a part of the successive capital adjustment. This produces a negative relationship between investment and the interest rate in accordance with Keynes' investment theory. This negative relationship holds not only for a basic shift in the interest rate but also for any type of shift and any original expected paths for the exogenous factors (Figure 6:7 and Prop. '3.3).

The assumption of an increasing supply curve for investment goods for the industry eliminates the apparent conflict between neoclassical and Keynes' invest-

ment theory.¹⁾

The long-run equilibrium solution in our model and the comparative statics result for this solution (Prop. 3.1 and 4.1) are in accordance with classical capital theory as developed by Böhm-Bawerk (1891), Wicksell (1901,1934) and Malinvaud (1953). Our investment theory and, hence, that of Keynes is therefore consistent with classical capital theory, when the latter is interpreted as a theory for long-run equilibrium positions only. Consequently, our investment theory encompasses Keynes' investment theory and much of the classical capital theory. The former being applicable to any expected paths for the exogenous factors and the latter being applicable to long-run equilibriums only. In addition, our model includes traditional neoclassical theory of the firm.

1) Lerner (1944) similarly assumes that the price of investment goods increases in aggregate investment in order to explain successive capital adjustment and a negative relation between investment and interest. As is revealed by Equation (3.11) in Section 3.8, Wijkman (1965), Lucas (1967b) and Uzawa (1969) derive a negative relation between investment and the interest rate given an increasing marginal cost curve for investment goods under stationary price expectations. Arvidsson (1960) also derives a negative relation between investment and the interest rate from an increasing supply curve for investment goods using a model in which future prices are treated as exogenous variables. Arvidsson's model is adequate for the industry only if investors believe that current investment has no effect on future prices. It is, therefore, open to the objection raised against the stationary expectations model in the introduction.

7. MONOPOLY AND NORMATIVE THEORY

7.1 Introduction

The preceding chapters analyze adjustment and investment determination in a competitive market. This chapter demonstrates that all earlier results are also applicable to a present value maximizing monopolist and to a normative model.

7.2 Monopoly7.2.1 Reinterpretation

This section demonstrates the applicability of all earlier results to a present value maximizing monopolist when the price functions for output and inputs are replaced by appropriate marginal revenue and marginal cost functions.

The monopolists optimization problem is as follows:

(7.1)

maximize

$\{X, I, L\}$

$$\int_{t_p}^{\infty} (P_X(X, t)X(t) - P_I(I, t)I(t) - P_L(L, t)L(t)) e^{-R(t, t_p)} dt$$

subject to

$$F(K, L, t) - X(t) \geq 0$$

$$\dot{K}(t) = I(t) - \delta K(t)$$

$$(K(t), L(t), X(t)) \geq 0$$

$$K(t_p) \text{ given.}$$

The necessary and sufficient conditions for an optimal solution are given by Equations (2.7)-(2.12) with the following substitutions:

1. $P_X(X, t)$ is replaced by the marginal revenue function:

$$(7.2) \quad MR(X, t) \triangleq \frac{d(P_X(X, t)X)}{dX} = P_X(X, t) \left(1 + \frac{1}{\epsilon_X(X, t)} \right),$$

where $\varepsilon_X(X,t)$ is the price elasticity of demand.

2. $P_I(I,t)$ and $P_L(L,t)$ are replaced by the marginal cost functions:

$$(7.3) \quad MCI(I,t) \stackrel{\Delta}{=} P_I(K,t) \left(1 + \frac{1}{\varepsilon_I(I,t)}\right)$$

$$(7.4) \quad MCL(L,t) \stackrel{\Delta}{=} P_L(L,t) \left(1 + \frac{1}{\varepsilon_L(L,t)}\right)$$

where ε_I and ε_L are the price elasticities of supply for the respective input.

The industry level optimization studied in Chapters 2-6 is then simply reinterpreted as the optimization problem for a monopolist. The demand function is replaced by the marginal revenue function and the supply functions are replaced by the marginal cost functions and corresponding assumptions are made for the new functions. Since the above analysis is based on Equations (2.7)-(2.12) we reach the remarkable conclusion that all earlier results are applicable to monopoly also.

For example, an expected upward shift in the marginal revenue function means higher investment from the time of information and successive capital adjustment for a monopoly (Prop. 3.3).¹⁾

Analogously, Proposition 6.1 gives the following comparative dynamics results for the effect of an upward shift in an exogenous factors under the conditions specified in the proposition:²⁾

$$(7.5) \quad \begin{array}{ccccccc} & + & & + & & + & - \\ I(t_p) = \varphi[MR(X,t), MCL(L,t), F'_K(K,L,t), F'_L(K,L,t), r(t) \text{ for} \\ & & & & & + & - \\ & t \geq t_p, MCI(I, t_p), MCI(I, t) \text{ for } t > t_p, K(t_p)] \end{array}$$

1) Nickell (1978, Ch. 3) draws similar conclusions. Our model is developed independently of Nickell.

2) The price elasticity of demand, ε , is reinterpreted as $\frac{\partial \ln X}{\partial \ln MR}$

Analogous results apply for the interpretation of investment determination as optimal capital adjustment towards a moving target, (Section 6.3), for the determinants of the degree of shortsighted investment, (Section 6.4), for the partial adjustment model, (Section 6.5), and for Keynes' investment theory, (Section 6.6).

7.2.2 Value_of_Earlier_Information

Some interesting differences do arise, however, concerning the impact of earlier information on a future shift in an exogenous factor. The monopolist would, of course, have an unchanged present value by sticking to the plans formulated without regard to the increase in the information lead. But this would be a suboptimal policy, since the unique optimal policy implies an adjustment at the earliest possible point in time (Prop. 3.3 and 4.2). Earlier information, therefore, always results in a higher discounted cash flow for a monopolist.

In a competitive market, however, earlier information on a shift that increases investment causes a lower discounted windfall capital gain (Prop. 3.4. Analogous results hold for a shift in $P_I(I)$). Earlier information has, therefore, an unfavorable impact on the group of capital owners! This occurs because competition forces the firms to select an industry level investment path that results in lower windfall gains, the earlier the information becomes available. This is an example of the well-known phenomenon that competition tends to decrease and eliminate gains.

Earlier information on a shift that increases investment produces a higher capital stock at each subsequent point in time. Consumers, therefore, gain through lower output prices (Prop. 3.4 and Lemma 2).

This produces the unconventional result that consumers and a monopolist have a common interest in earlier information on shifts that increase investment, while consumers and capital owners in a competitive market

have opposite interests in this question.¹⁾

7.3 Normative Planning Model

7.3.1 Reinterpretation

The model may also be reinterpreted as a normative model with a hypothetical social welfare maximizer, using different weights for benefits occurring to different groups in society.

Benefit from output is assumed to be measured as revenue plus consumer surplus as measured by the area between the compensated demand curve and output price with the assigned weight α :

$$\begin{aligned}
 (7.5) \quad \text{Benefit}(X,t) &\triangleq P_X(X,t) X(t) + \alpha \left[\int_0^{X(t)} P_X(Z,t) dZ - P_X(X,t) X(t) \right] \\
 &= (1-\alpha) P_X(X,t) X(t) + \alpha \int_0^{X(t)} P_X(Z,t) dZ
 \end{aligned}$$

Analogously, costs of inputs are measured as expenditures minus producer surplus as measured by the areas between the compensated supply curves and input prices with the assigned weights β and γ :

$$(7.6) \quad \text{CostI}(I,t) \triangleq P_I(I,t) I(t) - \beta \left[P_I(I,t) I(t) - \int_0^{I(t)} P_I(Z,t) dZ \right]$$

$$(7.7) \quad \text{CostL}(L,t) \triangleq P_L(L,t) L(t) - \gamma \left[P_L(L,t) L(t) - \int_0^{L(t)} P_L(Z,t) dZ \right]$$

The social welfare maximizers optimization problem then is:

1) Earlier information on a shift that decreases investment increases output price and decreases discounted windfall capital losses. Such earlier information is, therefore, unfavorable to consumers and favorable to capital owners under both monopoly and competition.

(7.8)

$$\text{Maximize } \int_{t_p}^{\infty} (\text{Benefit}(X,t) - \text{Cost}_I(I,t) - \text{Cost}_L(L,t)) e^{-R(t,t_p)} dt$$

$$\{X, I, L\}$$

subject to

$$\begin{aligned} F(K, L, t) - X(t) &\geq 0 \\ \dot{K}(t) &= I(t) - \delta K(t) \\ (K(t), L(t), X(t)) &\geq 0 \\ K(t_p) &\text{ given} \end{aligned}$$

The necessary and sufficient conditions for an optimal solution are given by Equations (2.7)-(2.12) with the following replacements:

1. $P_X(X, t)$ is replaced by the marginal benefit function

$$(7.9) \quad MB(X, t) \triangleq \frac{\Delta \text{Benefit}(X, t)}{\Delta X} = (1-\alpha) P_X(X, t) \left(1 + \frac{1}{\epsilon_X(X, t)}\right) + \alpha P_X(X, t)$$

2. $P_I(I, t)$ and $P_L(L, t)$ are replaced by the marginal cost functions:

$$(7.10) \quad MCI(I, t) \triangleq (1-\beta) P_I(I, t) \left(1 + \frac{1}{\epsilon_I(I, t)}\right) + \beta P_I(I, t)$$

$$(7.11) \quad MCL(L, t) \triangleq (1-\gamma) P_L(L, t) \left(1 + \frac{1}{\epsilon_L(L, t)}\right) + \gamma P_L(L, t)$$

The industry level optimization studied in Chapters 2-6 might, therefore, also be reinterpreted as a normative model for a social welfare maximizer, using different weights for different groups in society. If corresponding assumptions are made for the new functions, (7.9)-(7.11), we reach the conclusion that all earlier results are applicable to this case also.

In the presence of lump sum taxes, social welfare maximization requires $\alpha=\beta=\gamma=1$. The social welfare maximization model then provides the same solution as a competitive market. If consumer and producer surpluses are ignored, $\alpha=\beta=\gamma=0$. The social welfare maximizer then

behaves as a profit maximizing monopolist.¹⁾

7.3.2 Some Applications

The model might be applied to the analysis of optimal adjustment in a country's defence capacity in response to an expected shift in the demand for defence, induced perhaps by unexpected or expected changes in defence policies of other countries. More generally, the model determines the optimal defence policy for any expected paths for the marginal benefit curves for defence and the marginal cost curves for durable and nondurable inputs in the production of defence.

The model might also be interpreted as a normative theory of aggregate savings and capital for an entire economy. This suggests some extensions to the analysis of Arrow-Kurz (1970).

In this interpretation, the difference, $\text{Benefit}(X,t) - \text{CostI}(I,t)$, in the optimization model (7.8) is utility of consumption which is a function of aggregate output, X , and aggregate gross investment, I . This simplifies to $\text{Benefit}(X-I,t)$ for the Arrow-Kurz case with an one good economy. The marginal benefit function, $\text{MB}(X-I,t)$, is interpreted as the marginal utility of consumption and assumption 2, $\frac{\partial \text{MB}(X-I,t)}{\partial (X-I)} < 0$, means that the marginal utility of consumption is diminishing.

$\text{CostL}(L,t)$ is the disutility of labor. Marginal disutility, $\text{MCL}(L,t)$, is increasing in L . Finally $r(t)$ is the discount rate applicable to discount utilities at different points of time. The model determines the optimal path for aggregate gross savings, $\{I^*(t) \text{ for } t \geq t_p\}$, for any expected time paths for the exogenous factors.

Given this reinterpretation, the above results are applicable to the determination of the optimal path for

1) More generally, a social welfare maximizer should set α equal to $1/(1+m.e.b.)$, where m.e.b. is the marginal excess burden for public funds raised from consumers of X .

aggregate savings. The model implies, for example, that any expected temporary shift in population increases optimal savings at each point in time before the shift and increases optimal aggregate capital before as well as after the shift (Prop. 3.3).

Our extensions to the analysis of Arrow-Kurz are:

1. Arrow-Kurz study adjustment of inherited capital for constant exogenous factors only. This corresponds to our case with an unexpected shift introduced into an original stationary state (Section 3.3). The current study is applicable for any time path for the exogenous factors. Our model, therefore, allows for changing utility functions over time (due, for example, to population changes) and changing technology, $F(K,L,t)$. The Arrow-Kurz model is confined to constant growth rates for population and productivity involving constant growth deflated exogenous factors.
2. Unlike Arrow-Kurz, we include a variable volume of labor. Consequently, adjustment involves changes in the volume of labor in our model.
3. Our model allows for the case where utility depends separately on output and investment. The Arrow-Kurz model is confined to a one good economy, where utility depends only on the difference between output and investment. This extension is, however, of minor importance, since the linear form, $\text{Benefit}(X,t) - \text{CostI}(I,t)$, does not allow for any cross effects between X and I .

8. INVESTMENT FLUCTUATIONS

8.1 Introduction

This chapter demonstrates that the ex post path for investment tends to be quite volatile as a result of optimal capital adjustment (Section 8.2). We also examine the determinants of variability in investment (Section 8.3). Some hypotheses are derived offering interesting areas for future research.

8.2 Optimal or Excessive Adjustment

8.2.1 Introduction

Economic analyses and explanations of investment fluctuations are often based on the view that investors are too optimistic during favorable business conditions and too pessimistic during unfavorable conditions. Keynes' (1936, Ch. 22) explanation of the business cycles is, for example, largely based on the overadjustment of shortsighted investors. Similarly Hicks' (1950) and Goodwin's (1951, 1955) trade cycle models assume that investment is determined by a simple accelerator model until investors encounter unexpected upper or lower limits to investment. The market adjustment model assuming stationary expectations, (Lucas (1967b)), also implies overadjustment, as analyzed in Section 3.8. An additional example is given by the cobweb-model, interpreted as a model for capital adjustment with possible overadjustment (Clower (1954b) and Åberg (1968)).

This section shows, that the optimal ex post investment path tends to be quite volatile as a result of frequent changes in exogenous factors and new information in a changing world with uncertainty. Volatile investment may accordingly be interpreted as an indicator of optimal capital adjustment, rather than as an indicator of overadjustment.

8.2.2 Ex Ante and Ex Post Investment Paths Under Uncertainty

In a world with uncertainty new information typically arises quite frequently as the passage of time reveals the outcome of different types of random processes. A given ex ante investment path is, therefore, typically followed for a short period only. Thus, the ex post investment path is determined by relatively small portions of the ex ante paths. The thick curve in Figure 8:1 illustrates such an ex post investment path.¹⁾

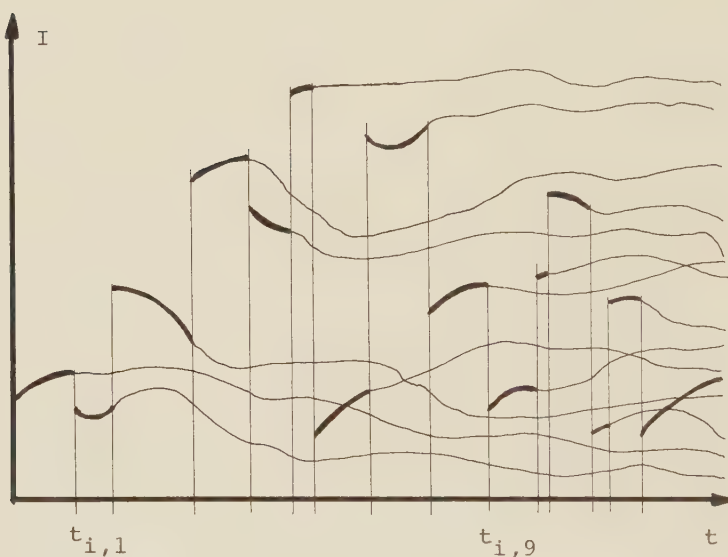


Figure 8:1. Ex ante and ex post investment paths.

Figure 8:1 reveals two sources of variation in ex post investment. Investment changes within an ex ante path due to expected changes in exogenous factors and capi-

1) This calls for an explicit analysis of uncertainty. A preliminary analysis was included in an earlier version of this study but has been excluded here due to limitations in space and time. The preliminary results, available upon request, suggest that this is an interesting area for further research. See also Hartman (1972, 1973) and Smith (1969, 1970).

tal over time. Actual investment also shifts between ex ante paths when new information emerges. In a changing world with a high degree of uncertainty on future changes in exogenous factors these two sources may give rise to quite volatile ex post investment. This holds for a competitive market, a monopoly and a social welfare maximizer.

The basic idea in our theory of investment for the industry is that investment is determined as a compromise between two counteracting factors; the desirability of having capital close to target capital and the desirability of having a smooth investment path. The optimal compromise typically involves considerable differences between planned and target capital and considerable investment variations even within an ex ante investment path. Concentration on the desirability of smooth investment involves excessive costs in the form of a capital path that differs excessively from the target capital path. The observed volatility in investment may be simply a result of the optimal compromise between these influences, rather than an indicator of overadjustment in investment. Similarly, a given capital path may appear to yield too slow capital adjustment with too large differences between actual and target capital. Focus on this factor only involves excessive costs of too extreme investment levels.

Earlier and more reliable information on the future development of the exogenous factors tends to result in smoother ex ante investment paths as well as less frequent and smaller shifts in investment when new information emerges (Prop. 3.4). This dampens the conflict between the desirability of having actual capital close to target capital and having smooth investment. The benefit of such information must, however, be weighed against the cost of its accumulation.¹⁾

1) For a recent survey on the theory of information see Hirshleifer and Riley (1979).

8.2.3 An Application

A simple example of the problem of optimal volatility of investment may be found in the shipping industry. The path for ship construction during the 70's should be seen in the light of the information available for investment decisions. The expansion of the stock in the late 60's and the early 70's may have been an optimal path given the available information. Our model predicts that the strong downward adjustment in expected future demand for shipping after the unexpected oil price increases in the early 70's would lead to subsequent downward shifts in investment and windfall capital losses for ship owners at the successive times of information, as analyzed in Chapter 3. Observed excessive shipping capacity with losses for ship owners and low levels of ship construction during the middle and late 70's are consistent, therefore, with an optimal adjustment to mainly unexpected changes.

The model also predicts that the level of ship construction eventually adjusts towards the new lower long-run equilibrium level of investment. This equals the rate of depreciation for basic capital plus the long-run growth rate times the new long-run equilibrium stock of ships. Expected changes in exogenous factors and the emergence of new information will, of course, prevent a smooth convergence towards the long-run equilibrium level. The notion of a long-run equilibrium may nevertheless provide a guide to the expected long-run development of a market.

The profitability of a shipyard should not, of course, be judged only on the basis of current prices during such an adjustment process. Instead investors in a competitive market should consider the entire expected price paths. In simple cases this involves an adjustment from current prices towards new long-run equilibrium prices. An investor in a market with imperfect competition considers instead the expected time path for the demand curve for the firm's output as analyzed in Chapter 7.

8.2.4 Empirical Testing of Optimal Adjustment Versus Overadjustment

The above model of investment determination suggests that the observed volatility of investment may be explained as optimal adjustment to expected changes in exogenous factors and to frequent emergence of new information on the future development of exogenous factors. This is an alternative to the explanation of overadjustment contained in the studies referred to. We now discuss an empirical test of these two hypotheses, although testing lies outside the scope of this study. The ex post profitability of a marginal investment may be measured by

$$\int_t^{\infty} V(K,s) e^{-R(s,t) - \delta(s-t)} ds - P_I(I,t),$$

for ex post values for functions and variables. For the overadjustment hypothesis one may obtain the testable prediction that ex post profitability is relatively low for investment decisions made under favorable business conditions. The state of business conditions might be measured from factors such as the investment level, output price and the degree of capacity utilization. A significant negative relationship between ex post profitability and these proxies would support the overadjustment hypothesis. A weak or a positive relationship would provide no support for the hypothesis.

Under the additional assumption of rational expectations about developments for the exogenous factors our investment model for optimal adjustment predicts that the ex post profitability of the marginal investment shows no systematic relationship to information available to investors at the time of the investment decision. An insignificant relation between ex post profitability and business conditions at the time of the decision would support the optimal adjustment hypothesis, while a strong negative or positive relation would not.

The outcomes of these tests have important policy imp-

lications. If the overadjustment hypothesis is supported for certain types of investment, this is a highly valuable piece of information for investors. It then would be profitable for investors to decrease investment under favorable business conditions and increase investment under unfavorable business conditions relative to earlier policy. Such a policy adjustment would typically give rise to smoother investment and an efficiency gain from a social welfare point of view.

A test of the hypothesis of overadjustment is also interesting since this hypothesis provides an argument for governmental action intended to stabilize investment.

Another interesting application is in a comparative analysis of the performance of different types of organizations taking investment decisions. The absence of a systematic relationship between ex post profitability and different types of information available at the time of the investment decision indicates a well designed investment policy.

The results for the relative performance of the public versus the private sector and labor-managed versus capitalist-managed firms have important implications for the debate on the desirability of different types of organization for decision-making. The performance of a competitive market versus a market with one dominating agent might shed light on the importance of motivation and on the significance of Darwinistic selection within a competitive market relative to the single firm's advantages on forecasting, planning and coordination. Results might have relevance to an evaluation of these market structures.

These empirical tests lie outside the scope of the current study, but appear to offer interesting avenues for future research.

8.3 Determinants of Investment Variability

8.3.1 Introduction

This section examines the determinants of the variability in optimal ex post investment.

8.3.2 Investment Variations for a Given Change in Exogenous Factors

A high rate of depreciation has two counteracting effects on investment variations. First, it leads to high replacement investment and, hence, a low percentage change in gross investment for a given change in net investment. High replacement investment, thus, tends to absorb investment changes that arise from expected changes in exogenous factors and new information. For the case with a basic disturbance that yields an increase in capital of $\theta\%$ within a year, the percentage change in investment for that year is approximately:

$$(8.1) \quad \frac{\Delta I}{I^+} = \frac{\theta K^+}{\delta K^+} = \frac{\theta}{\delta}$$

Since δ might vary from, say, .01 to 1 for different types of capital, (8.1) indicates much stronger investment variations for long-lived as compared to short-lived capital.

This effect is, however, counteracted by the earlier result that a high rate of depreciation induces fast capital adjustment. Interpreted in Equation (8.1), a higher δ produces a higher percentage change in capital within a year, θ , for a given change in the exogenous factor.

To examine the relative strength of these factors and the importance of other determinants of investment variations, consider the effect of a one percent basic upward shift in the marginal revenue product function, (i.e., $V_1(K) = 1.01 V_0(K)$). A linear approximation of Equation (6.3) around the original stationary state produces the following approximation to the shift in

target capital:

$$(8.2) \quad \frac{K_1^+ - K_0^+}{K_0^+} \approx \frac{.01}{(P_I^+ - e1. + |V^+ - e1.|)}$$

An unexpected shift of this type produces approximately the following shift in investment at the time of the shift, using the partial adjustment model, (Section 6.5):

$$(8.3) \quad \frac{\Delta I}{I_0^+} = \frac{n(K_1^+ - K_0^+)}{\delta K_0^+} = .01 \frac{\sqrt{\left(\frac{r}{2}\right)^2 + (r+\delta)\delta(1 + (|V^+ - e1.|/P_I^+ - e1.))}}{\delta[P_I^+ - e1. + |V^+ - e1.|]} - \frac{r}{2}$$

Figure 8:1 illustrates the percentage shift in investment for different values for the relevant variables.

Observations and conclusions from Figure 8:1:

1. A higher rate of depreciation brings about smaller changes in investment. This means that the stabilizing effect of higher replacement investment dominates the counteracting tendency to faster capital adjustment. The model predicts, for example, larger variations in building than in short-lived machinery (*ceteris paribus*).
2. If the substitution effect balances the output effect (i.e., if $(\sigma + \epsilon) = 0$), a one percent unexpected shift in demand involves a one percent shift in the marginal revenue product function (Lemma A.3.ii). For the parameter values considered in Figure 8:1 this produces a 0.5-1 percent shift in investment. Since the increase in output is dampened by a price increase along the short-run marginal cost curve, a reasonable output increase by 0.2-0.5% arises together with an investment change that is one to five times as strong as the output change. Unexpected demand shifts, therefore, tend to induce much stronger percentage changes in investment than in output. This result is explained by the stock-flow relationship between capital and investment, where capital is adjusted

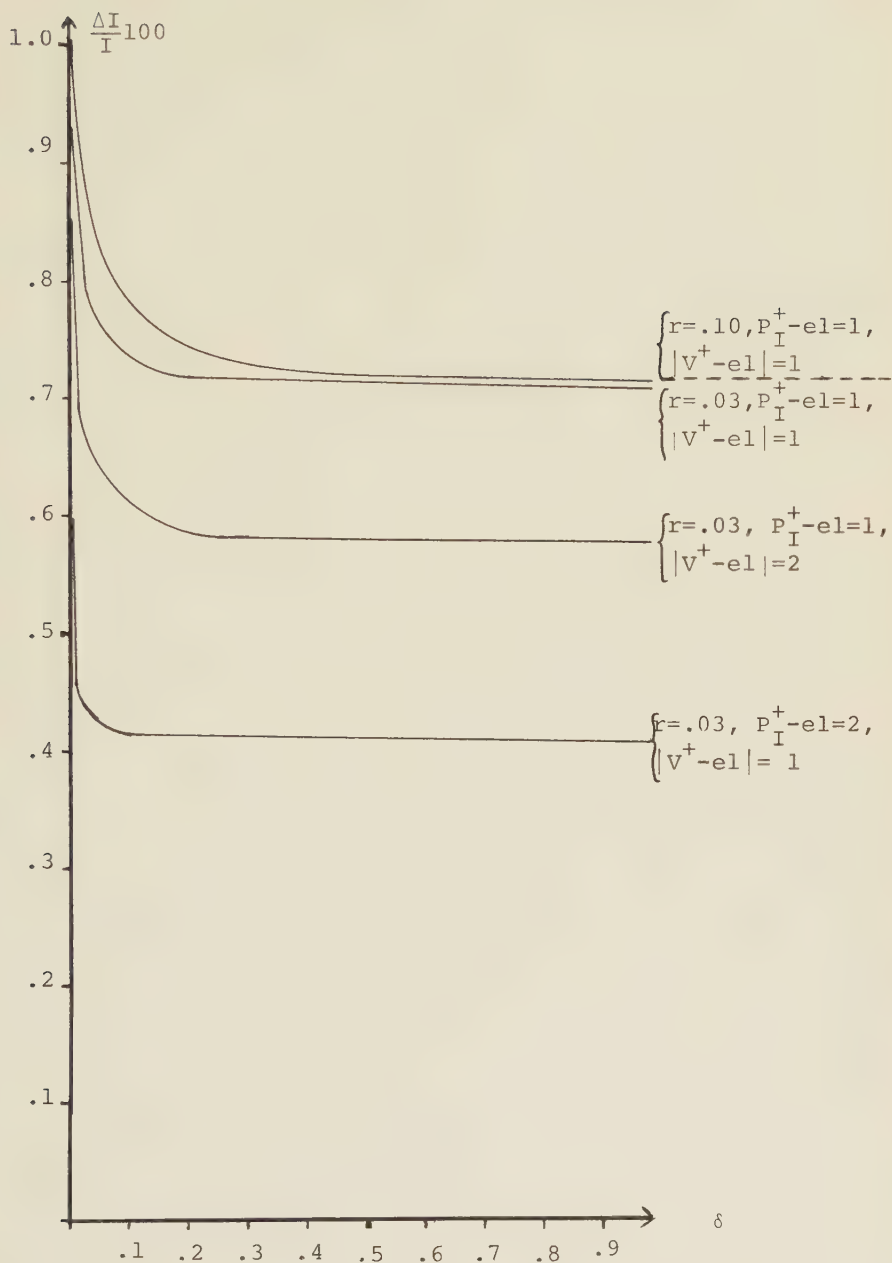


Figure 8:1. Percentage shift in investment for a one percentage unexpected shift in the marginal revenue product function.

through a flow of investment. A given change in capital within some period often requires a much stronger percentage change in investment, especially in the case of long-lived capital.

3. The interest rate has approximately no effect on variability in stationary state capital (Eq. (8.2)). A higher r causes, however, larger investment variations as a result of faster capital adjustment, (Section 3.7). Figure 8:1 shows that this factor is quite weak for reasonable ranges of the real rate of interest.
4. A higher $|V^{+el.}|$ produces lower variability in stationary state capital, (Eq. (8.2)), and counteracting faster capital adjustment (Section 3.7). For the parameter ranges considered in Figure 8:1 the former effect dominates, producing more stable investment for a higher $|V^{+el.}|$.
5. A higher P_I -elasticity produces lower variability in stationary state capital, (Eq. (8.2)), and reinforcing slower capital adjustment. A double P_I -elasticity almost eliminates half of the shift in investment in Figure 8:1.

The degree of investment variations appears to be mainly determined by the rate of depreciation, the $|V\text{-elasticity}|$ and the elasticity of the supply curve for investment goods. The first two elements may be interpreted as the major determinants of the degree of variability in "demand" for investment. The P_I -elasticity distributes a given change in demand into I and P_I -changes.

The model predicts, therefore, that the demand for houses varies more than the demand for clothes. The high rate of depreciation for clothes means high replacement investment, so that a given change in net investment is absorbed with a relatively small percentage change in gross investment. The low geographic mobility of houses contributes to a steep supply curve. Consequently, variations in demand lead to relatively small variations in the construction of houses and re-

latively large variation in house prices. The higher mobility of clothes tends to produce a flatter supply curve and, hence, relatively large I -variations and small P_I -variations.

An industry producing for a domestic market with a low absolute price elasticity of demand faces a high absolute V -elasticity and, consequently, relatively stable investment as compared to an industry producing for an international market with a high absolute price elasticity of demand. The steep demand curve produces a stable target capital and consequently stable investment in spite of the counteracting faster adjustment.

8.3.3 A Lower Long-Run Growth Rate

The rate of depreciation in the above model is the sum of the depreciation rate for basic capital and the long-run growth rate, as analyzed in 3.2.3. A lower long-run growth rate gives rise to more volatile investment, with the strongest effect on long-lived capital, (Figure 8:1). The outcome of a lower long-run growth rate is lower replacement investment for growth deflated capital, $(\delta_b + g_1)K$, and also, therefore, lower absorption. A given change in growth deflated capital then requires a higher percentage change in gross investment. The lower growth rate for GNP in most developed countries during the 70's and the lower expected growth rate for the 80's as compared to earlier decades are, therefore, predicted to yield more volatile investment, especially for long-lived capital. The lower growth rate results in a more severe conflict between the desirability of smooth investment and the desirability of fast capital adjustment. To some extent, this may explain the appeal of a high growth rate. A low growth rate during the 80's is predicted to produce not only low demand for investment but also a relatively high variability in this demand with serious consequences for producers of buildings, ships, trucks, cars and long-lived machinery and equipment.

8.3.4 Variability in Exogenous Factors

The variability of investment also depends, of course, on the variability of exogenous factors. Large expected changes in exogenous factors as well as large and frequent revisions in expectations when new information emerges bring about volatile ex post investment.

Larger variations in the demand for the services of clothes than in the demand for housing services tend to bring about larger variations in the production of clothes than in house construction (*ceteris paribus*).

As another application, consider the following chain of stock-flow markets: the market for shipping services, the market for ships and the market for shipyard equipment. A given change in demand for shipping services that results in some change in the volume of shipping services will have a stronger effect on the volume of ship construction and an even stronger impact on the production of shipyard equipment.

Analogously, a given change in demand for preschooling exerts a stronger impact on the demand for preschool teachers and a still stronger impact on the demand for teachers to train preschool teachers.

The effect on investment prices in later links in the chain is typically mitigated by a high degree of substitution among outputs of different kinds of investment goods, i.e. a flat supply curves obtain for certain types of investment goods. A decrease in demand for shipyard equipment could be met by an increase in the production of other types of equipment with strong effects on the volume but fairly limited effects on the price of shipyard equipment.

8.3.5 Empirical Testing

The above analysis may be used to predict and explain the variability in volumes and prices of different types of investment goods. An empirical examination of these

factors lies outside the scope of the current study but appears to be an interesting area for future research.

APPENDIX

Proof of Proposition 2.1 (p. 41):

Since condition ii) for an equilibrium coincides with (2.5) we only need to show that an optimal solution for each firm for the price paths $\{P_X^*(t), P_I^*(t), P_L^*(t)\}$ is equivalent to an optimal solution for the industry level optimization problem (2.6) for these price paths.

Let $\{X^{i,*}(t), I^{i,*}(t), L^{i,*}(t)\}$ be an optimal solution for firm i . To prove that the associated aggregate paths $\{\Sigma X^{i,*}(t), \Sigma I^{i,*}(t), \Sigma L^{i,*}(t)\}$ produce an optimal solution to (2.6) assume the contrary, i.e. assume there exists a feasible solution $\{X^Z(t), I^Z(t), L^Z(t)\}$ with the feasible paths for firm i $\{X^{i,Z}(t), I^{i,Z}(t), L^{i,Z}(t)\}$ where

$$\int_{t_p}^{\infty} [P_X^*(t) (\Sigma X^{i,Z} - \Sigma X^{i,*}) - P_I^*(t) (\Sigma I^{i,Z} - \Sigma I^{i,*}) - P_L^*(t) (\Sigma L^{i,Z} - \Sigma L^{i,*})] e^{-R(t, t_p)} dt > 0$$

This implies:

$$\int_{t_p}^{\infty} [P_X^*(t) (X^{i,Z} - X^{i,*}) - P_I^*(t) (I^{i,Z} - I^{i,*}) - P_L^*(t) (L^{i,Z} - L^{i,*})] e^{-R(t, t_p)} dt > 0$$

for at least one firm. This contradiction proves that separate maximizations produces an optimal solution to (2.6).

For the proof in the other direction, let $\{X^*(t), I^*(t), L^*(t)\}$ be a solution to (2.6) with the associated implicit price of capital service, $P_K^*(t)$.

From (2.14), (2.15'), (2.17) and (2.19):

$$(A.1) \quad P_X^* F'_K(K^*, L^*) = P_K^* \quad P_X^* F'_L(K^*, L^*) = P_L^*$$

From the Euler equation, $F'_K K + F'_L L = X$ and (A.1):

$$(A.2) \quad X^* P_X^* - K^* P_K^* - L^* P_L^* = 0$$

From assumption 5:

$$(A.3) \quad (X - X^*)P_X^* - (K - K^*)P_K^* - (L - L^*)P_L^* \leq 0 \text{ for all } X \in F(K, L)$$

From (2.15) and (2.19):

$$(A.4) \quad P_I^*(t) = \int_{t_p}^{\infty} P_K^*(s) e^{-\delta(s-t) - R(s,t)} ds.$$

$$\text{Since } K(t) - K^*(t) = \int_{t_p}^t (I(s) - I^*(s)) e^{-\delta(t-s)} ds,$$

$$\begin{aligned} (A.5) \quad & \int_{t_p}^{\infty} (K(t) - K^*(t)) P_K^*(t) e^{-R(t, t_p)} dt = \\ & \int_{t_p}^{\infty} \int_{t_p}^t (I(s) - I^*(s)) e^{-\delta(t-s)} P_K^*(t) e^{-R(t, t_p)} ds dt = \\ & = \int_{t_p}^{\infty} (I(s) - I^*(s)) P_I^*(s) e^{-R(s, t_p)} ds \end{aligned}$$

From (A.3) and (A.5):

$$\begin{aligned} (A.6) \quad & \int_{t_p}^{\infty} [(X(t) - X^*(t)) P_X^*(t) - (I(t) - I^*(t)) P_I^*(t) - \\ & (L(t) - L^*(t)) P_L^*(t)] e^{-R(t, t_p)} dt \leq 0 \end{aligned}$$

for all feasible paths $\{X(t), I(t), L(t)\}$.

Using this result we next show that the path for the i :th firm in the solution to (2.6), $\{X^{i,*}(t), I^{i,*}(t), L^{i,*}(t)\}$, is an optimum for this firm. Assuming the contrary, i.e.:

$$(A.7)$$

$$\begin{aligned} & \int_{t_p}^{\infty} ((X^i(t) - X^{i,*}(t)) P_X^*(t) - (I^i(t) - I^{i,*}(t)) P_I^*(t) - (L^i(t) - L^{i,*}(t)) \\ & P_L^*(t)) e^{-R(t, t_p)} dt > 0 \end{aligned}$$

for some feasible solution $\{X^i(t), I^i(t), L^i(t)\}$. Then $\{X^*(t) + X^i(t) - X^{i,*}(t), I^*(t) + I^i(t) - I^{i,*}(t), L^*(t) + L^i(t) - L^{i,*}(t)\}$ is a feasible solution for the industry, producing a contradiction between (A.6) and (A.7). This completes the proof of the Proposition.

Lemma A.1: The solution for the equilibrium price and quantity paths is unique.

Proof: Assume there are two different equilibrium price paths,

$$\{P_{X,0}^*(t), P_{I,0}^*(t), P_{L,0}^*(t)\} \text{ and}$$

$$\{P_{X,1}^*(t), P_{I,1}^*(t), P_{L,1}^*(t)\}$$

with associated quantity paths indexed 0 and 1 respectively. Then

$$\int_{t_p}^{\infty} (P_{X,0}^*(X_0^* - X_1^*) - P_{I,0}^*(I_0^* - I_1^*) - P_{L,0}^*(L_0^* - L_1^*)) e^{-R(t, t_p)} dt \geq 0$$

and

$$\int_{t_p}^{\infty} (P_{X,1}^*(X_1^* - X_0^*) - P_{I,1}^*(I_1^* - I_0^*) - P_{L,1}^*(L_1^* - L_0^*)) e^{-R(t, t_p)} dt \geq 0,$$

This implies

(A.8)

$$\begin{aligned} & \int_{t_p}^{\infty} ((P_{X,1}^* - P_{X,0}^*)(X_1^* - X_0^*) - (P_{I,1}^* - P_{I,0}^*)(I_1^* - I_0^*) - \\ & (P_{L,1}^* - P_{L,0}^*)(L_1^* - L_0^*)) e^{-R(t, t_p)} dt \geq 0 \end{aligned}$$

Since $P_X(X, t)$ is strictly decreasing in X ,

$(P_{X,1}^* - P_{X,0}^*)(X_1^* - X_0^*) \leq 0$. Analogously, since $P_I(I, t)$ and $P_L(L, t)$ are strictly increasing in I and L respectively, $(P_{I,1}^* - P_{I,0}^*)(I_1^* - I_0^*) \geq 0$ and $(P_{L,1}^* - P_{L,0}^*)(L_1^* - L_0^*) \geq 0$.

Since the price paths differ, some of these inequalities hold strictly for some time interval. This contradicts (A.8) and proves that the solution for the equilibrium price paths is unique. Since $P_X(X, t)$ is strictly increasing in X while $P_I(I, t)$ and $P_L(L, t)$ are strictly decreasing in I and L respectively, this implies a unique solution for the quantity paths also. This completes the proof of the Lemma.

Proof of Lemma 1 p. 43: Differentiation of (2.13) gives:

$$(A.9) \quad \frac{dG}{dK} = - \frac{P'_X F'_K F'_L + P_X F''_{KL}}{P'_X F'^2_L + P_X F''_{LL} - P'_L}$$

The production function, $F(K,L)$, is homogeneous of degree one. This implies that the associated marginal product functions, $F'_K(K,L)$ and $F'_L(K,L)$, are homogeneous of degree zero. The Euler equations for these functions are:

$$(A.10) \quad F'_K K + F'_L L = X$$

$$(A.11) \quad F''_{KK} K + F''_{KL} L = 0, \quad F''_{LK} K + F''_{LL} L = 0$$

From (2.14) and (A.9)-(A.11):

$$\begin{aligned} (A.12) \quad \frac{\partial V(K,t)}{\partial K} &= F'_K P'_X (F'_K + F'_L \frac{dG}{dK}) + P_X (F''_{KK} + F''_{KL} \frac{dG}{dK}) \\ &= [-F'_K P'_X (F'_K P'_L + F'_L P_X F''_{KL} - F'_K P_X F''_{LL}) + P_X (F''_{KK} P'_X F'^2_L - F''_{KL} P'_L \\ &\quad - F''_{KL} P'_X F'_K F'_L)] / (P'_X F'^2_L + P_X F''_{LL} - P'_L) < 0, \quad (\text{since } F'_K > 0, F'_L > 0, \\ &\quad F''_{KL} > 0, F''_{KK} < 0, F''_{LL} < 0, P'_X < 0, P'_L > 0). \end{aligned}$$

This completes the proof of the Proposition.

Lemma A.2: The necessary and sufficient conditions

(2.7)-(2.12) are equivalent to (2.15)-(2.18) or (2.15')-(2.18).

Proof: From (2.8), (2.12) and assumptions 3 and 6:

$$\lim_{t \rightarrow \infty} \lambda_2(t) = 0$$

From Equations (2.7), (2.10) and (2.14) we obtain:

$$\begin{aligned} \lambda_2(t) &= - \int_t^{\infty} (\dot{\lambda}_2(s) - \delta \lambda_2(s)) e^{-\delta(s-t)} ds = \\ &= \int_t^{\infty} V(K,s) e^{-R(s,t_p) - \delta(s-t)} ds, \end{aligned}$$

which together with (2.8) implies (2.15). Differentiation of (2.15) with respect to time gives (2.15').

Noting that output price is strictly positive by assumption 2, (2.16)-(2.18) follow directly from (2.9)-(2.12).

For the proof in the other direction, note that shadow prices defined as $\lambda_1(t) = P_X(X, t)e^{-R(t, t_p)}$ and $\lambda_2(t) = P_I(I, t)e^{-R(t, t_p)}$ combined with (2.16)-(2.18) lead to (2.8)-(2.12). If (2.15') is added, the fact that $\lambda_2(t) = (P_I' + \frac{\partial P_I}{\partial t} - r(t)P_I)e^{-R(t, t_p)}$ produces (2.7). If instead (2.15) is added we obtain (2.15'). We can then proceed to (2.7). This completes the proof.

Proof of Lemma 2 p. 45:

From (2.18), (A.9) and (A.11):

$$\frac{dX}{dK} = F_K' + F_L' \frac{dG}{dK} = \frac{-1}{P_X' F_L'^2 + P_X'' F_{LL}'' - P_L'} (-F_K' P_X' F_{LL}'' + F_K' P_L' + F_L' P_X' F_{KL}'')$$

> 0 , proving i.

Since F is homogeneous of degree one: $F(K, L) = L \cdot F(K/L, 1)$

Let $k \triangleq K/L$ and $f(k) \triangleq F(K/L, 1)$. Then since $F(K, L) = L \cdot f(K/L)$, we have:

$$F_K' = f_k', \quad F_L' = f - k f_k', \quad F_{KL}'' = -f_{kk} k^{1/L}$$

From the definition of the elasticity of substitution, σ :

$$(A.13) \quad \frac{1}{\sigma} \triangleq - \frac{d(F_K'/F_L')}{dk} \frac{k}{F_K'/F_L'} = \frac{F_{KL}'' F(K, L)}{F_K' F_L'}$$

From (A.9) and (A.13) we then have:

$$\begin{aligned} \frac{dL}{dK} &= \frac{-P_X' F_{KL}'' X}{P_X' F_L'^2 + P_X'' F_{LL}'' - P_L'} \left(\frac{F_K' F_L'}{F_{KL}'' X} + \frac{1}{P_X'} \frac{P_X}{X} \right) \\ &= \frac{-P_X' F_{KL}'' X}{P_X' F_L'^2 + P_X'' F_{LL}'' - P_L'} (\sigma + \epsilon) \stackrel{<}{>} 0 \text{ when } (\sigma + \epsilon) \stackrel{>}{<} 0, \end{aligned}$$

where ϵ is the price elasticity of demand. This completes the proof.

Proof of Lemma 3 p. 45:

Investment is finite, since in the opposite case assumptions 1, 3, 5 and 8 combined with (2.14) contradict (2.15).

Since investment and, therefore, the price of investment goods is finite, assumptions 2 and 5, the continuous capital path and (2.15) produce a finite marginal revenue product in an optimal solution. Equation (2.15') then leads to the conclusion that investment changes are finite and, hence, that planned investment is continuous except possibly for points in time when $\frac{\partial P_I(I,t)}{\partial t}$ is infinite for some I . This completes the proof of the Lemma.

Lemma A.3 (Shifts in basic functions):

- i. $P_{X,1}(X) > P_{X,0}(X)$ for each $X \Rightarrow V_1(K) > V_0(K)$ for each K
- ii. $P_{L,1}(L) > P_{L,0}(L)$ for each $L \Rightarrow V_1(K) > (<) V_0(K)$ for each K if the sum of the elasticity of substitution and the price elasticity of demand, $\sigma + \epsilon$, > 0 (< 0).
- iii. $F'_{K,1}(K,L) > F'_{K,0}(K,L)$ for each $(K,L) \Rightarrow V_1(K) > (<) V_0(K)$ for each K if

$$[P_X F''_{LL}(1+\epsilon) + \frac{P_X F'_L F'_L}{X} - P'_L \left(-\frac{KF'_K}{X} + \epsilon\right)] > 0 \text{ } (< 0)$$
- iv. $F'_{L,1}(K,L) > F'_{L,0}(K,L)$ for each $(K,L) \Rightarrow V_1(K) < (>) V_0(K)$ for each K if $(1+\sigma+\epsilon + (F'_K P'_L / F''_{KL} P_X)) > 0$ (< 0).

Proof: Let the shift in demand be represented by a shift parameter $\alpha \in [0,1]$, such that $P_X(X,\alpha) \triangleq$

$(1-\alpha)P_{X,0}(X,t) + \alpha P_{X,1}(X,t)$. Consequently:

$$P'_{X\alpha} \triangleq \frac{\partial P_X(X,\alpha)}{\partial \alpha} > 0. \text{ Analogously } P'_{L\beta} \triangleq \frac{\partial P_L(L,\beta)}{\partial \beta} > 0,$$

$F''_{K\gamma} \triangleq \frac{\partial F'_K(K,L,\gamma)}{\partial \gamma} > 0$ and $F''_{L\eta} \triangleq \frac{\partial F'_L(K,L,\eta)}{\partial \eta} > 0$. Let Z denote the change in the marginal revenue product of capital for a given capital, $\bar{K}$. From (2.14), (2.17) and (A.10) we then have:

$$\begin{aligned}
 (A.14) \quad & P_X(X, \alpha) F'_K(\bar{K}, L, \gamma) - V(\bar{K}) - Z = 0 \\
 & P_X(X, \alpha) F'_L(\bar{K}, L, \eta) - P_L(L, \beta) = 0 \\
 & X - K F'_K(\bar{K}, L, \gamma) - L F'_L(\bar{K}, L, \eta) = 0
 \end{aligned}$$

Total differentiation of (A.14) using (A.11) gives:

$$\begin{pmatrix} P'_X F'_K & P_X F''_{KL} & -1 \\ P'_X F'_L & P_X F''_{LL} - P'_L & 0 \\ 1 & -F'_L & 0 \end{pmatrix} \begin{pmatrix} dX \\ dL \\ dZ \end{pmatrix} =
 \begin{pmatrix} -P'_{X\alpha} F'_K & 0 & -P_X F''_{K\gamma} & 0 \\ -P'_{X\alpha} F'_L & P'_{L\beta} & 0 & -P_X F''_{L\eta} \\ 0 & 0 & K F''_{K\gamma} & L F''_{L\eta} \end{pmatrix} \begin{pmatrix} d\alpha \\ d\beta \\ d\gamma \\ d\eta \end{pmatrix}$$

Denoting the first matrix A, we have

$$|A| = P'_X F'_L F'_L + P_X F''_{LL} - P'_L < 0 \quad (\text{since } P'_X < 0, F'_L > 0, P_X > 0, F''_{LL} < 0, P'_L > 0).$$

Using Cramer's rule:

$$\begin{aligned}
 \frac{dZ}{d\alpha} &= - \frac{\begin{vmatrix} P'_X F'_K & P_X F''_{KL} & -P'_{X\alpha} F'_K \\ P'_X F'_L & P_X F''_{LL} - P'_L & -P'_{X\alpha} F'_L \\ 1 & -F'_L & 0 \end{vmatrix}}{|A|} = \\
 &= \frac{(P_X F''_{LL} - P'_L) P'_{X\alpha} F'_K - P_X F''_{KL} P'_{X\alpha} F'_L}{|A|} > 0, \text{ proving i.}
 \end{aligned}$$

$$\begin{aligned}
 \frac{dZ}{d\beta} &= \frac{\begin{vmatrix} P'_X F'_K & P_X F''_{KL} & 0 \\ P'_X F'_L & P_X F''_{LL} - P'_L & P'_{L\beta} \\ 1 & -F'_L & 0 \end{vmatrix}}{|A|} = \frac{P'_{L\beta} P_X F''_{KL} X(\sigma + \epsilon)}{|A|} > 0 \quad (< 0)
 \end{aligned}$$

if $(\sigma + \epsilon) > 0 (< 0)$, proving ii.

$$\begin{aligned}
 \frac{dZ}{d\gamma} &= \frac{\begin{vmatrix} P'_X F'_K & P_X F''_{KL} & -P_X F''_{K\gamma} \\ P'_X F'_L & P_X F''_{LL} - P'_L & 0 \\ 1 & -F'_L & K F''_{K\gamma} \end{vmatrix}}{|A|} = \\
 &= \frac{F''_{K\gamma} P'_X X [P_X F''_{LL} (1 + \epsilon) + \frac{P_X F'_L F'_L}{X} - P'_L \cdot (-\frac{K F'_K}{X} + \epsilon)]}{|A|} > 0 \quad (< 0) \text{ if}
 \end{aligned}$$

the parenthesis is positive (negative), proving iii.

$$\frac{dz}{d\eta} = \frac{\begin{vmatrix} P_X' F_K' & P_X' F_{KL}'' & 0 \\ P_X' F_L' & P_X' F_{LL}'' - P_L' & -P_X' F_{L\eta}'' \\ 1 & -F_L' & L F_{L\eta}'' \end{vmatrix}}{|A|} =$$

$$= \frac{-F_{L\eta}'' P_X' F_{KL}'' P_X' X[1+\sigma+\epsilon+(F_K' P_L' / F_{KL}'' P_X' X)]}{|A|} < 0 \text{ } (> 0) \text{ if the}$$

parenthesis is > 0 (< 0). This completes the proof of the Lemma.

Proof of Proposition 3.1 p. 58:

Introducing the additional shift parameter:

$$P_{I\theta}' = \frac{\partial P_I(I, \theta)}{\partial \theta} > 0,$$

(2.15'), (2.17) and (A.10) produce the following conditions for a stationary state:

$$(A.15) \quad \begin{cases} P_X(X, \alpha) F_K'(K, L, \gamma) - (r+\delta) P_I(\delta K, \theta) = 0 \\ P_X(X, \alpha) F_L'(K, L, \eta) - P_L(L, \beta) = 0 \\ X - K F_K'(K, L, \gamma) - L F_L'(K, L, \eta) = 0 \end{cases}$$

By total differentiation of (A.15), using (A.11):

(A.16)

$$\begin{pmatrix} P_X' F_K' & P_X' F_{KK}'' - (r+\delta) \delta P_I' & P_X' F_{KL}'' \\ P_X' F_L' & P_X' F_{LK}'' & P_X' F_{LL}'' - P_L' \\ 1 & -F_K' & -F_L' \end{pmatrix} \begin{pmatrix} dX \\ dK \\ dL \end{pmatrix} =$$

$$\begin{pmatrix} -P_{X\alpha}' F_K' & 0 & -P_X' F_{K\gamma}'' & 0 & (r+\delta) P_{I\theta}' & P_I' \\ -P_{X\alpha}' F_L' & P_{L\beta}' & 0 & -P_X' F_{L\eta}'' & 0 & 0 \\ 0 & 0 & K F_{K\gamma}'' & L F_{L\eta}'' & 0 & 0 \end{pmatrix} \begin{pmatrix} d\alpha \\ d\beta \\ d\gamma \\ d\eta \\ d\theta \\ dr \end{pmatrix}$$

Denoting the first matrix B and using the implication from (A.11) $F''_{KL}F''_{LK}=F''_{KK}F''_{LL}$, we conclude that $|B| > 0$.

Using Cramer's rule:

$$\frac{dX}{d\alpha} = \frac{\begin{vmatrix} -P'_{X\alpha}F'_K & P_XF''_{KK} - (r+\delta)\delta P'_I & P_XF''_{KL} \\ -P'_{X\alpha}F'_L & P_XF''_{LK} & P_XF''_{LL} - P'_L \\ 0 & -F'_K & -F'_L \end{vmatrix}}{|B|} > 0$$

$$\frac{dK}{d\alpha} = \frac{\begin{vmatrix} P'_X F'_K & -P'_{X\alpha} F'_K & P_X F''_{KL} \\ P'_X F'_L & -P'_{X\alpha} F'_L & P_X F''_{LL} - P'_L \\ 1 & 0 & -F'_L \end{vmatrix}}{|B|} > 0$$

$\frac{dL}{d\alpha} > 0$ by symmetry for K and L in (A.15).

Since $F'_K(K,L)$ and $F'_L(K,L)$ are homogeneous of degree zero, the marginal products are determined by the quotient K/L only. F'_K is strictly decreasing and F'_L is strictly increasing in this quotient. F'_K or F'_L must therefore increase at least weakly. Since both K and L increase strictly P'_I and P'_L increase strictly. (A.15) then means that P_X increases, i.e. $\frac{dP_X}{d\alpha} > 0$. This completes the proof of the Proposition.

Proof of Proposition 3.2 p. 62:

Assume that the (K,I) path does not converge to the stationary state point for $t \geq t_f$, (K^+, I^+) , determined by (3.3) and (3.4). Since the $\dot{I}=0$ -curve is constant over time after time t_f , Equations (3.3)-(3.4) reveal that eventually, at time $t_g (> t_f)$ say, the future (K,I) -path is fully in the interior of the northeast or the southwest region as compared to (K_1^+, I_1^+) in Figure 3:7, i.e., $K(t) > K_1^+$ and $I(t) > I_1^+$ for each $t \geq t_g$ or $K(t) < K_1^+$ and $I(t) < I_1^+$ for each $t \geq t_g$. Since

$$(A.17) \quad (r(t_f) + \delta)P_I(I_1^+, t_f) = V(K_1^+(t_f), t_f), \quad P'_I > 0, \text{ and } V' < 0,$$

this implies that the l.h.s. and the r.h.s. in the

following equation, derived from (2.15), have opposite signs:

$$(A.18) \quad P_I(I(t_g), t_f) - P_I(I_1^+(t_f), t_f) \\ = \int_{t_g}^{\infty} [V(K(s), t_f) - V(K_1^+(t_f), t_f)] e^{-(r(t_f) + \delta)(s - t_g)} ds.$$

This contradiction proves the Proposition.

Proof of Proposition 3.3 p. 71:

Let indexes 0 and 1 denote functions and optimal paths for the original and the new time paths for the exogenous factors.

To prove $K_1^* > K_0^*$, assume the contrary, i.e. $K_1^* \leq K_0^*$ for some interval, starting at say time t_a . This is illustrated for two cases in Figure A:1.

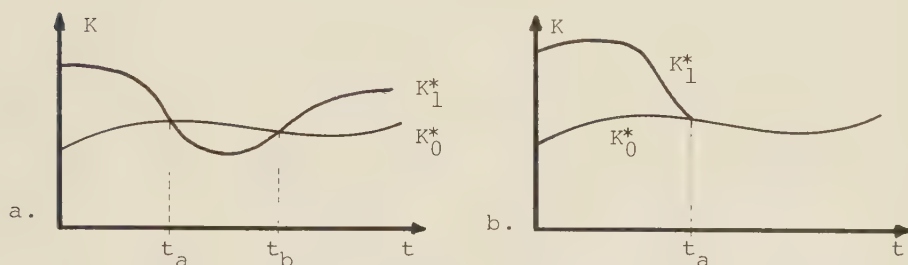


Figure A:1. Impossible paths for K_1^* .

Since $I_1^*(t_a) \leq I_0^*(t_a)$, (2.15) implies:

(A.19)

$$0 > P_I(I_1^*, t_a) - P_I(I_0^*, t_a) = \\ \int_{t_a}^{\infty} [(V(K_1^*, s) e^{-R(s, t_a)})_1 - (V(K_0^*, s) e^{-R(s, t_a)})_0] e^{-\delta(s - t_a)} ds$$

Since an upward shift in $V(K, s)$ and a downward shift in $r(s)$ produce an upward shift in the discounted V -function, $V(K, s) e^{-R(s, t_a)}$, the integrand is positive as long as K_1^* is less than K_0^* . The inequality, therefore, means that K_1^* , eventually, say at time t_b , is strictly higher than K_0^* , in case $I_1^*(t_b) > I_0^*(t_b)$ and $P_I(I_1^*, t_b) -$

$P_I(I_0^*, t_b) > 0$, or that $K_1^*(t) = K_0^*(t)$ for each $t \geq t_a$. These cases are illustrated in Figure A:1 a and b respectively. In the latter case the upward shift in the discounted V-function on the r.h.s. of (A.19) means that K_1^* must have been strictly higher than K_0^* for some time before t_a , implying that (A.19) holds with strict inequality. Hence, in any case, for an interval $[t_a, t_b]$ where $K_1^* \leq K_0^*$:

$$(A.20) \quad P_I(I_1^*, t_a) - P_I(I_0^*, t_a) \\ - [P_I(I_1^*, t_b) - P_I(I_0^*, t_b)] e^{-R(t_b, t_a) - \delta(t_b - t_a)} < 0$$

From (2.15), (A.20) is equal to:

$$\int_{t_a}^{t_b} [(V(K_1^*(s), s) e^{-R(s, t_a)})_1 \\ - (V(K_0^*(s), s) e^{-R(s, t_a)})_0] e^{-\delta(s - t_a)} ds.$$

Since $K_1^* \leq K_0^*$ in this interval and the discounted V-function shifts upwards, this integral is positive, contradicting (A.20). This means that $K_1^*(t) > K_0^*(t)$ for each $t > t_i$, proving i.

Let $t_c \in [t_i, t_s]$ and let common values for variables and functions for paths 0 and 1 be unindexed. We then have:

$$(A.21) \quad 0 < P_I(I_1^*, t_i) - P_I(I_0^*, t_i) = \\ \int_{t_i}^t [V(K_1^*(s), s) - V(K_0^*(s), s)] e^{-R(s, t_i) - \delta(s - t_i)} ds + \\ e^{-\delta(t_c - t_i)} \left(\int_{t_c}^{\infty} [(V(K_1^*(s), s) e^{-R(s, t_i)})_1 \right. \\ \left. - (V(K_0^*(s), s) e^{-R(s, t_i)})_0] e^{-\delta(s - t_c)} ds \right. \\ \left. + \int_{t_i}^{t_c} [V(K_1^*(s), s) - V(K_0^*(s), s)] e^{-R(s, t_i) - \delta(s - t_i)} ds \right. \\ \left. + e^{-R(t_c, t_i) - \delta(t_c - t_i)} (P_I(I_1^*, t_c) - P_I(I_0^*, t_c)) \right),$$

where the inequality follows from the fact that I_1 must be strictly higher than I_0 at time t_i to give a strictly higher K immediately after time t_i .

Since K_1 is strictly higher than K_0 after time t_i , the next to the last term in (A.21) is negative. The last term is, therefore, strictly positive, implying $I_1^* > I_0^*$ for $t \in [t_i, t_s]$ and proving ii.

Moreover (A.21) shows that a strictly positive windfall capital gain arise through the strictly higher investment at time t_i proving iii.

Finally, let $t_d \in [t_e, \infty[$. We then have:

(A.22)

$$P_I(I_1^*, t_d) - P_I(I_0^*, t_d) = \int_{t_d}^{\infty} (V(K_1^*, s) - V(K_0^*, s)) e^{-R(s, t_d) - \delta(s - t_d)} ds$$

Since K_1^* is strictly higher than K_0^* , the last term is strictly negative, and, therefore, I_1^* is strictly lower than I_0^* for any t_d in the interval $[t_e, \infty[$. This completes the proof of the Proposition.

Proof of Proposition 3.4 p. 73:

Let indexes 0, 1 and 2 denote functions and variables in the absence of a shift and for shifts with information leads $(t_s - t_{i,1})$ and $(t_s - t_{i,2})$ respectively.

Proposition 3.3 implies $K_2^*(t) > K_1^*(t)$ for $t \in [t_{i,2}, t_{i,1}]$, since K_1^* follows K_0^* while K_2^* exceeds K_0^* for this interval. Analogous to the proof of Proposition 3.3, this implies $K_2^*(t) > K_1^*(t)$ for each $t > t_{i,2}$. Thus we arrive at proof of i.

Moreover, Proposition 3.3 implies $I_2^*(t) > I_1^*(t)$ for $t \in [t_{i,2}, t_{i,1}[$, since I_1^* follows I_0^* while I_2^* exceeds I_0^* for this interval. This proves ii.

From (2.15):

$$(A.24) \quad P_I(I_2^*, t) - P_I(I_1^*, t) = \int_t^{\infty} [(V(K_2^*, s) e^{-R(s, t)})_2 - (V(K_1^*, s) e^{-R(s, t)})_1] e^{-\delta(s - t)} ds$$

for $t \geq t_{i,1}$. Since the discounted V-functions are identical and $K_2^*(s) > K_1^*(s)$ for $s \geq t_{i,1}$, this integral is strictly negative and, thus, $I_2^*(t) < I_1^*(t)$ for $t \geq t_{i,1}$. This proves iii.

Discounted windfall capital gain at time $t_{i,2}$ may be written as:

(A.25)

$$\begin{aligned}
 & (P_I(I_2^*, t_{i,2}) - P_I(I_0^*, t_{i,2})) e^{-R(t_{i,2}, 0)} \\
 &= \int_{t_{i,2}}^{t_{i,1}} [(V(K_2^*, s) e^{-R(s, 0)})_2 - (V(K_0^*, s) e^{-R(s, 0)})_0] e^{-\delta(s - t_{i,2})} ds \\
 &+ \int_{t_{i,1}}^{\infty} [(V(K_2^*, s) e^{-R(s, 0)})_2 - (V(K_0^*, s) e^{-R(s, 0)})_0] e^{-\delta(s - t_{i,2})} ds \\
 &< \int_{t_{i,1}}^{\infty} [(V(K_1^*, s) e^{-R(s, 0)})_1 - (V(K_0^*, s) e^{-R(s, 0)})_0] e^{-\delta(s - t_{i,2})} ds \\
 &= [P_I(I_1^*, t_{i,1}) - P_I(I_0^*, t_{i,1})] e^{-R(t_{i,1}, 0)} e^{-\delta(t_{i,1} - t_{i,2})},
 \end{aligned}$$

The equalities follow from (2.15). Since discounted V-functions indexed 0 and 2 merge for $[t_{i,2}, t_{i,1}]$ and $K_2^* > K_0^*$ for this interval, the first term after the equality sign is negative and could be dropped without violating the inequality sign. Moreover, the discounted V-functions indexed 1 and 2 merge for $t \geq t_{i,1}$ and $K_2^* > K_1^*$, so that a change from K_2^* to K_1^* strictly increases the value of the marginal product, in accordance with the inequality sign. Since $e^{-\delta(t_{i,1} - t_{i,2})} < 1$, the discounted windfall capital gain at time $t_{i,1}$ exceeds the last term while (A.25) in turn states that the last term exceeds the discounted windfall capital gain at time $t_{i,2}$. An earlier time of information, therefore, produces a lower discounted and, hence, a lower undiscounted windfall capital gain. This proves iv.

To examine the limiting case with perfect foresight, note that discounted windfall capital gain is:

$$\begin{aligned}
 (A.26) \quad & (P(I_1^*, t_i) - P(I_0^*, t_i)) e^{-R(t_i, 0)} = \\
 & = \int_{t_i}^{\infty} [(V(K_1^*, s) e^{-R(s, 0)})_1 - (V(K_0^*, s) e^{-R(s, 0)})_0] e^{-\delta(s-t_i)} ds.
 \end{aligned}$$

As t_i approaches minus infinity, the discounted V-functions merge except for times where $(s-t_i) \rightarrow \infty$. Moreover, the r.h.s. approaches zero as $(s-t_i)$ approaches infinity. This means that K_1 approaches K_0 for all finite $(s-t_i)$ and, consequently, that discounted windfall capital gain approaches zero as t_i approaches minus infinity. This completes the proof of the Proposition.

Proof of Proposition 4.1 p. 100:

Applying Cramer's rule on (A.16) we have:

$$\frac{dK}{d\beta} = \frac{\begin{vmatrix} P'_{XK} & 0 & P'_{XKL} \\ P'_{XL} & P'_{L\beta} & P'_{XLL} - P'_L \\ 1 & 0 & -F'_L \end{vmatrix}}{|B|} = \frac{-P'_{L\beta} P'_{XKL} (\sigma + \epsilon)}{|B|} > 0 \quad (< 0)$$

if $(\sigma + \epsilon) > 0 \quad (< 0)$.

$$\begin{aligned}
 \frac{dL}{d\beta} &= \frac{\begin{vmatrix} P'_{XK} & P'_{XKK} - (r + \delta) \delta P'_I & 0 \\ P'_{XL} & P'_{XLK} & P'_{L\beta} \\ 1 & - & -F'_K \end{vmatrix}}{|B|} = \\
 &= \frac{P'_{L\beta} (P'_{XKK} - (r + \delta) \delta P'_I + P'_{XK} F'_K)}{|B|} < 0
 \end{aligned}$$

$$\begin{aligned}
 \frac{dX}{d\beta} &= \frac{\begin{vmatrix} 0 & P'_{XKK} - (r + \delta) \delta P'_I & P'_{XKL} \\ P'_{L\beta} & P'_{XLK} & P'_{XLL} - P'_L \\ 0 & -F'_K & -F'_L \end{vmatrix}}{|B|} = \\
 &= \frac{P'_{L\beta} (-F'_K P'_{XKL} + (P'_{XKK} - (r + \delta) \delta P'_I) F'_L)}{|B|} < 0
 \end{aligned}$$

Since X decreases for an unchanged demand curve, P_X in-

creases, proving 1.

Analogously:

$$\frac{dK}{d\gamma} = \frac{\begin{vmatrix} P'_{XK} & -P_{XF''_{KY}} & P_{XF''_{KL}} \\ P'_{XL} & 0 & P_{XF''_{LL}} - P'_L \\ 1 & KF''_{KY} & -F'_L \end{vmatrix}}{|B|} =$$

$$= -F''_{KY} P'_{XK} [P_{XF''_{LL}} (1+\epsilon) + \frac{P_{XF'_{LL}}}{X} - P'_L \left(\frac{F'_K}{X} + \epsilon \right)] / |B|$$

>0 (<0) if the parenthesis is >0 (<0).

$$\frac{dL}{d\gamma} = \frac{\begin{vmatrix} P'_{XK} & P_{XF''_{KK}} - (r+\delta) \delta P'_I & -P_{XF''_{KY}} \\ P'_{XL} & P_{XF''_{LK}} & 0 \\ 1 & -F'_K & KF''_{KY} \end{vmatrix}}{|B|} =$$

$$= \frac{F''_{KY} P'_{XK} P_{XF''_{KL}} P_X [1 + \sigma + \epsilon + (F'_K (r+\delta) \delta P'_I / F''_{KL} P_X X)]}{|B|}$$

<0 (>0) if the parenthesis is >0 (<0).

$$\frac{dX}{d\gamma} = \frac{\begin{vmatrix} -P_{XF''_{KY}} & P_{XF''_{KK}} - (r+\delta) \delta P'_I & P_{XF''_{KL}} \\ 0 & P_{XF''_{LK}} & P_{XF''_{LL}} - P'_L \\ KF''_{KY} & -F'_K & -F'_L \end{vmatrix}}{|B|} =$$

$$= F''_{KY} [P_X P_{XF''_{KL}} F'_L - K P'_L P_{XF''_{KK}} - K (r+\delta) \delta P'_I (P_{XF''_{LL}} - P'_L) -$$

$$P_{XF'_K} (P_{XF''_{LL}} - P'_L)] / |B| > 0$$

Since X increases for an unchanged demand curve, P_X decreases. This proves 2.

Finally 3 holds by symmetry to 2, while 4 holds by symmetry to 1. This completes the proof of the Proposition.

Proof of Proposition 4.2 p. 104:

To prove $K_1^* > K_0^*$, assume the contrary, i.e. $K_1^*(t) \leq K_0^*(t)$ for some $t \in [t_i, t_s]$. Analogous to the proof of Proposition 3.3 this means $K_1^*(t) \leq K_0^*(t)$ for each $t \in [t_i, t_s]$, so that $I_1^*(t_i) \leq I_0^*(t_i)$ and hence $P_I(I_1^*, t_i) - P_I(I_0^*, t_i) \leq 0$.

From (2.15)

(A.27)

$$[P_{I,1}(I_1^*, t_c) - P_{I,0}(I_0^*, t_c)] e^{-\delta(t_c - t_i) - R(t_c, t_i)} = \\ [P_I(I_1^*, t_i) - P_I(I_0^*, t_i)] - \int_{t_i}^{t_c} (V(K_1^*, s) - V(K_0^*, s)) e^{-\delta(s - t_i) - R(s, t_i)} ds$$

Since the r.h.s. is negative as long as K_1^* is less than K_0^* , this implies that I_1^* is less than I_0^* and thus that K_1^* remains less than K_0^* for each $t > t_i$. The upward shift in the P_I -function implies that K_1^* is strictly lower than K_0^* for some interval after time t_s .

Since

(A.28)

$$P_I(I_1^*, t_i) - P_I(I_0^*, t_i) = \int_{t_i}^{\infty} (V(K_1^*, s) - V(K_0^*, s)) e^{-\delta(s - t_i) - R(s, t_i)} ds,$$

the lower K_1^* implies that (A.28) is strictly positive. This contradicts the earlier opposite conclusion and proves i.

This result gives $I_1^*(t_i) > I_0^*(t_i)$. Since P_I is increasing in I , this proves iii.

Let $t_a \in [t_i, t_s[$. From (2.15):

$$0 < P_I(I_1^*, t_i) - P_I(I_0^*, t_i) = \\ t_a \int_{t_i}^a (V(K_1^*, s) - V(K_0^*, s)) e^{-R(s, t_i) - \delta(s - t_i)} ds \\ + (P_I(I_1^*, t_a) - P_I(I_0^*, t_a)) e^{-R(t_a, t_i) - \delta(t_a - t_i)}$$

Since the first term on the r.h.s. is negative, the

last term is strictly positive, so that $I_1^*(t_a) > I_0^*(t_a)$, completing the proof of the Proposition.

Proof of Proposition 6.1 p. 116:

Results 1, 2, 4, 6 and 7 follow from Lemma A.3, Proposition 3.3 and Proposition 4.2. To prove 3 note that the contrary assumption, $I_1^*(t_p) \geq I_0^*(t_p)$, means $K_1^*(t) \geq K_0^*(t)$ for each $t > t_p$ in analogous manner to the proof of Proposition 3.3. From (2.15):

(A.29)

$$P_{I,1}(I_1^*, t_p) - P_{I,0}(I_0^*, t_p) =$$

$$\int_{t_p}^{\infty} (V(K_1^*, s) - V(K_0^*, s)) e^{-\delta(s-t_p) - R(s, t_p)} ds$$

The upward shift in P_I makes the l.h.s. strictly positive, while the higher K_1^* makes the r.h.s. negative. This contradiction proves 3.

To prove 8, note that $K_1(t_p) > K_0(t_p)$ means $K_1^*(t) > K_0^*(t)$ for each $t \geq t_p$, so that $P_I(I_1^*, t_p) < P_I(I_0^*, t_p)$ from (A.29), and $I_1^*(t_p) < I_0^*(t_p)$. This completes the proof of the Proposition.

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